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Counterpart of Advanced TS discrete controller without matrix inversion

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Abstract: This paper aims to present a systematic methodology for designing a Counterpart of an Advanced Takagi-Sugeno (CATS) discrete controller. Advance controllers for nonlinear systems under Takagi-Sugeno representation have been designed for years using efficient control laws such as the non-PDC (Parallel Distributed Compensation) controller. In terms of stabilization, that kind of controllers is a powerful tool which allows outperforming the classical PDC results using non quadratic Lyapunov function. However, in spite of these advantages, this control strategy presents a major inconvenient from real time implementation point of view: the use of a nonlinear matrix inversion at each sample time. In order to solve this problem, our paper presents a CATS controller design methodology to obtain an equivalent to the non-PDC controller without matrix inversion. Through a given procedure associated with a stability analysis, not only the efficiency is proved but also its validity. Finally, some simulation results will emphasize the originality and the usefulness of the proposed CATS controller.

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1. INTRODUCTION

In past years, facing nonlinearity in dynamical systems have become less and less a huge obstacle. Instead of the standard linearization that appeared in industry, some powerful tools have been developed. One of them is the Takagi-Sugeno (TS) representation (Takagi and Sugeno, 1985) which can be expressed as a polytopic model based on a combination of several linear models linked together with nonlinear terms. Obtained from a nonlinear system based on a sector nonlinearity approach, a TS representation has the advantage of being an exact equivalence of the nonlinear system on a defined set.

The stability and stabilization of closed-loop systems is mainly studied using Lyapunov functions. In (Guerra and Vermeiren, 2004), the authors present a first case, the quadratic Lyapunov function, and, secondly, the non-quadratic one also introduced in discrete-time. Non-quadratic Lyapunov functions have also been studied for continuous-time in (Guerra *et al.*, 2012).

In order to control nonlinear systems, lots of methods have been developed. Among them, the Parallel Distributed Compensation (PDC) controller developed in (Wang *et al.*, 1996) allows including nonlinearity inside a classic state or output feedback controller. Used in plenty of works, it can be however generalized in a more powerful tool called non-PDC controller presented, for example, in (Guerra and Vermeiren, 2004). As developed along this paper, the combination of non-quadratic Lyapunov function and non-PDC controller gives less conservative results.

For many years, the non-PDC controller has been used through plenty of works and papers. (Ding *et al.*, 2006) propose in their work an extension to the previous results of (Guerra and Vermeiren, 2004). (Bouarar *et al.*, 2009) have studied the particular case of static output feedback for descriptors systems using a non-PDC control law. One year after, (Bouarar *et al.*, 2010) focus on robustness with H_∞ conditions. (Mozelli *et al.*, 2010) based their work on descriptors systems, Finsler's lemma and so on to get new relaxations and less conservativeness thanks to a non-PDC controller. (Xie *et al.*, 2013) construct a new non-PDC control law structure for discrete-time systems. In the last papers of (Lendek *et al.*, 2013; Lendek *et al.*, 2015), the theory of TS controllers has been extended, using periodic Lyapunov functions and delayed non-quadratic Lyapunov functions. In the recent works of (Laurain *et al.*, 2015a) and (Laurain *et al.*, 2015b), the non-PDC structure is used for periodic observers instead of control.

In both cases discrete and continuous time, the non-PDC controller, associated with a non-quadratic Lyapunov function, implies a matrix inversion each time you reconsider the control law, i.e., each clock time of the embedded computer that calculates the control input. For matrices of big size, such as ten squares and ten columns, this matrix inversion every sampling time can be time consuming, taking into account the limited capacities of some real-time embedded hardware. Even if non-PDC controllers offer useful and powerful results, they can be difficult to implement this limitation.

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As a conclusion of the state-of-the-art, non-PDC structure have been hugely used in the literature, in order to improve controllers or observers efficiency, develop new relaxations and reduce conservativeness for an important family of systems (continuous-time, discrete-time, robust systems, disturbed-by-noise systems…). However, in spite of its efficiency, non-PDC structure presents the inconvenient of the matrix inversion. This paper proposes a method for finding an equivalency to a non-PDC control law for a discrete-time system, called CATS controller.

It is organized following the structure: Some preliminaries are presented to introduce the context, including a motivating example. The main contribution is detailed in Section 3 with the new control law, the choice of the Lyapunov function and the stability analysis. Finally, Section 4 emphasizes the efficiency of the proposed methodology by presenting some simulation results.

2. PRELIMINARIES

Keeping in mind that our purpose is real applications, we will mainly focus on discrete-time because the embedded computer that calculates the control law is triggered at every sample time.

2.1 Notation

Along this paper, the following notations are used:

$$A_z = \sum_{i=1}^{r} h_i\left(z(t)\right)A_i \,,\; A_{zz} = \sum_{i=1}^{r} h_i\left(z(t)\right)h_j\left(z(t)\right)A_{ij}$$

$$A_{zz-} = \sum_{i=1}^{r} h_i\left(z(t)\right)h_j\left(z(t-1)\right)A_{ij} \,,$$

$$A_{zz+} = \sum_{i=1}^{r} h_i\left(z(t)\right)h_j\left(z(t+1)\right)A_{ij}$$

2.2 Motivating example

For this paper, we base this work on the previous study of the literature, (Guerra and Vermeiren, 2004). In their paper, they propose an example that has been considered in many other papers as a good academic example for nonlinear research, such as (Nguyen *et al.*, 2015).

The nonlinear system is based on the following matrices:

$$\begin{gathered} A_1 = \begin{bmatrix} 1 & -\beta \\ -1 & -0.5 \end{bmatrix}, B_1 = \begin{bmatrix} 5+\beta \\ 2\beta \end{bmatrix} \\ A_2 = \begin{bmatrix} 1 & \beta \\ -1 & -0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 5-\beta \\ -2\beta \end{bmatrix} \end{gathered} \tag{1}$$

For a given β, it is proved that a quadratic Lyapunov function and a PDC controller cannot stabilize this system which is not so complex (only 2 rules and square matrices of size 2). However, for a given β, the use of a non-PDC control law is mandatory. Moreover, for highly complex real systems such as Internal Combustion (IC) engines, an efficient control needs powerful controllers. As a conclusion for the motivating example, some nonlinear and complex systems can be controlled only by advanced control laws such as non-PDC controllers, even if this implies matrices inversion and high computational cost. Consequently, there is a need for an equivalent control law that avoids the matrix inversion.

2.3 Non-quadratic Lyapunov function and non-PDC controller conditions

For this paper, we base this work on the conditions for a new non-quadratic Lyapunov function and a non-PDC controller such as the ones developed in (Guerra and Vermeiren, 2004):

$$\begin{cases} u(t) = -F_z H_z^{-1} x \\ V(t) = x^T H_z^{-T} P_z H_z^{-1} x \end{cases} \tag{2}$$

Applying the Lyapunov stability method with such a controller and the previously detailed system, the authors define the following quantity:

$$\Gamma_{ij}^{k} = \begin{bmatrix} P_i & (*) \\ A_i H_j - B_i F_j & H_k + H_k^T - P_k \end{bmatrix} \tag{3}$$

In order to make the system (1) stable, the previous quantity must check the following conditions:

$$\Gamma_{ii}^{k} < 0 \,,\; i,k \in \{1,\ldots,r\} \tag{4}$$

$$\frac{2}{r-1}\Gamma_{ii}^{k} + \Gamma_{ij}^{k} + \Gamma_{ji}^{k} < 0 \,,\; i,j,k \in \{1,\ldots,r\}, i \neq j \tag{5}$$

This leads to a Linear Matrix Inequality (LMI) problem such as defined in (Boyd et al., 1994). This problem can be solved by the LMI Toolbox of Matlab, which can provide a solution and, consequently, the gains of the non-PDC controller.

3. MAIN CONTRIBUTION

3.1 Control law approximation with multiple sums

In this subsection, we present the main contribution: the procedure to develop an equivalent control law to the non-PDC controller, based on a Counterpart of this Advanced Takagi-Sugeno (CATS) controller. As detailed in the preliminaries, we base the original contribution on non-PDC controller for discrete time (Guerra and Vermeiren, 2004).

By this way, the first step of the approximation process is to solve the LMI problem defined in (3) in order to get the gains of the non-PDC controller. To explain the principle, let us consider the following non-PDC control law taken from (2):

$$u(k) = -F_z H_z^{-1} x(k) \tag{6}$$

The objective is to find an equivalent controller which involves non inversion of weighted matrices. To this purpose,

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