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Actuator Fault diagnosis: H_∞ framework with relative degree notion

D. Ichalal * B. Marx ** D. Maquin ** J. Ragot **

* Evry Val d'Essonne University, IBISC-Lab, 40, rue de Pevoux, 91020, Evry Courcouronnes, France (email: dalil.ichalal@ibisc.univ-evry.fr

** CRAN UMR 7039, Université de Lorraine, CNRS, 2 avenue de la forêt de Haye, TSA TSA 60604, 54518 Vandoeuvre-les-Nancy Cedex, France

Abstract: A new actuator fault diagnosis and estimation approach is proposed for dynamical systems. The main contribution consists in enhancing the fault detection with a new observer that takes into account the relative degree of the output of the system with respect to the fault. The Single Input Single Fault (SIFO) case is considered to present the approach and an extension to systems with multiple outputs and multiple faults. The convergence of the proposed residual generator is analyzed using the Lyapunov theory which can be expressed straightforwardly in terms of Linear Matrix Inequalities (LMIs). Numerical examples are provided in order to illustrate the proposed approach.

Keywords: Linear Systems, Actuator Fault Diagnosis, Relative Degree, H_∞ framework, LMIs.

1. INTRODUCTION

These last years, the problem of fault diagnosis in dynamical systems has taken an important place in engineering control. This is due to the great demand in high performance control systems even in the presence of faults. For example, in industrial production and transportation fields, the systems should operate without time discontinuity in order to meet demand and to avoid delays that may cause considerable loss of money. Furthermore, systems must be designed to ensure the safety of the human operator and system itself such as in aeronautics and aerospace vehicles. The components of the system (actuators, sensors,...) may be affected by a fault which can cause some unpleasant repercussions on the system and the human operator. Therefore, the control systems should be designed by taking into account the faulty scenarios. Some results have been reported in the context of fault diagnosis (fault detection, isolation and estimation) using different approaches: namely, signal processing, model-based, soft computing approaches,..., (Patton et al. [2001],Gertler [1998],Frank and Ding [1997]). They aim to monitor the system and provide some information to be used in the control task to compensate the faults and preserve the healthy system functioning with adequate fault tolerant controllers.

In the context of model-based fault diagnosis, the H_∞ approach is one of the most interesting techniques in designing residual generators that provide fault indicators and allow to detect, isolate and estimate the magnitude of the faults. The presence of disturbances and measurement noises may affect the problem of fault detection and isolation, by masking the effect of the fault, which causes a delay in the detection of the fault and hence may lead to disaster consequences. Therefore, the H_∞ framework has been extended by using the H_- performance index in order to enhance the sensitivity of the fault on the residual signal. Several interesting results are reported in the literature. In Chadli et al. [2013], the H_-/H_∞ has been exploited in order to generate residual signals that are made sensitive as possible for the faults and less sensitive as possible to the disturbances and the measurement noises. In Ichalal et al. [2014], the min/max problem is transformed into a simple minimization problem by introducing a filter that aims to enhance the sensitivity of the residual signal to the fault and minimizes the effect of the disturbances. In Mazars et al. [2008], a reference model is used in order to shape the response of the residual signal. An adequate choice of the reference model can significantly enhance the sensitivity of the residual signal to the fault.

After this brief bibliography, it has been noted that, in general, the same faults affecting the state equation affect also the output equation. This commonly used assumption allows to guarantee the regularity assumption needed in the H_∞ framework, which lead to good performances of the residual generators. However, in real systems, actuator faults are different of the sensor ones, then, the regularity assumption is not satisfied, hence, degraded performances of the residual generator are obtained which affect the fault sensitivity. A solution to this issue is given in Ichalal et al. [2014] by perturbing the output of the system with the actuator faults and a small parameter. Acceptable performances are then obtained regarding to the fault sensitivity, but these performances depend on the fixed small parameter.

In this paper, a new solution for the problem of actuator fault detection and isolation is proposed by using the relative degree notion. This work is a continuation of the result given in Ichalal et al. [2014]. The use of relative

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degree notion aims to define new auxiliary outputs depending on the actuator faults. Indeed, by differentiating the outputs of the system according to the relative degrees with respect to the faults, new outputs can be generated and the system with the new output vector satisfies the regularity condition. Of course, the implementation of the proposed approach is based on time derivatives of the noisy outputs which are obtained by the recent robust algorithms that provides high order time derivatives with good convergence properties and insensitivity to measurement noises. For example, one can cite the High Order Sliding Mode Differentiator (HOSMD) having a finite time convergence property Levant [2003], the Non-Asymptotic algebraic differentiator in Fliess et al. [2008], the Linear Time Varying differentiator in Ibrir [2003] and the High gain differentiator in Kalsi et al. [2010]. In this work, the HOSMD is used. Before these advances in noisy signal differentiation, several works are proposed for fault diagnosis by using time derivatives of the output as in Hou and Patton [1998] which are related to the system dynamics left invertibility.

Throughout the paper, the following definitions and lemmas will be used.

Definition 1. Relative Degree. Isidori [1995] Consider the linear system

$$\dot{x}(t) = Ax(t) + Ef(t), \quad y(t) = Cx(t) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $f \in \mathbb{R}$ is the fault signal and $y \in \mathbb{R}$ is the output signal. The relative degree of the system (1) is the number r satisfying

$$\begin{cases} CA^{i-1}E = 0, \forall i = 1, ..., (r-1) \\ CA^{r-1}E \neq 0 \end{cases} \tag{2}$$

In other words, the relative degree corresponds to the number of successive time derivatives of the output to obtain an equation that involves the fault signal f

$$y^{(r)}(t) = CA^r x(t) + \underbrace{CA^{r-1}E}_{\neq 0} f(t) \tag{3}$$

Definition 2. Bounded Real Lemma. Boyd et al. [1994] For the system

$$\dot{x}(t) = Ax(t) + Ef(t), \; y(t) = Cx(t) + Df(t) \tag{4}$$

The system (4) is stable and satisfy the conditions

$$\begin{cases} \lim\limits_{t \to +\infty} y = 0 & \text{if } f = 0 \\ \|y(t)\|_2 < \gamma \|f\|_2 & \text{if } f \neq 0 \end{cases} \tag{5}$$

if the following LMI is satisfied

$$\begin{pmatrix} A^T P + PA & PE & C^T \\ E^T P & -\gamma I & D^T \\ C & D & -\gamma I \end{pmatrix} < 0 \tag{6}$$

with P, a symmetric and positive definite matrix. Furthermore, if $\gamma > 0$ is minimized, the transfer from f to y is also minimized.

2. PROBLEM STATEMENT AND MOTIVATION

Let us consider the linear system subject to an actuator fault

$$\begin{cases} \dot{x}(t) = Ax(t) + Ef(t) \\ y(t) = Cx(t) \end{cases} \tag{7}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $f(t) \in \mathbb{R}$ is the actuator fault and $y(t) \in \mathbb{R}$ is the system output. The matrices A, E and C are real constant with appropriate dimensions.

Note that without loss of generality, the control input of the system is omitted (i.e. $u(t) = 0$), the extension to systems having $u(t) \neq 0$ is straightforward. Assume that the output $y(t)$ has a relative degree r with respect to the fault $f(t)$ and the system is observable (i.e. the pair (C, A) is observable).

Classically, an H_∞ Residual Generator is described by the following equations

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + L(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \\ r(t) = M(y(t) - \hat{y}(t)) \end{cases} \tag{8}$$

without taking into account the relative degree of the system. The matrices L and M are real constant to be determined (For the Single Fault and Single Output system, M is reduced to a scalar variable). In order to make the residual $r(t)$ sensitive to the fault $f(t)$, a virtual residual $r_e(t)$ can be defined by the equation

$$r_e(t) = r(t) - f(t) \tag{9}$$

Let us define the state estimation error $e(t) = x(t) - \hat{x}(t)$. The system generating the virtual residual signal $r_e(t)$ is then described as follows

$$\begin{cases} \dot{e}(t) = (A - LC)\, e(t) + Ef(t) \\ r_e(t) = MCe(t) - f(t) \end{cases} \tag{10}$$

Consequently, the gain matrices L and M should be designed in such a way to minimize the effect of $f(t)$ with respect to $r_e(t)$. It is then clear that if $r_e(t) \to 0$, the real residual signal $r(t)$ tends to the fault $f(t)$ i.e. $r(t) \to f(t)$ which allows to detect the fault $f(t)$ (note that if $r_e(t) = 0$, we have $r(t) = f(t)$ which provides the fault estimation, this is the ideal case which cannot be achieved by the H_∞ approach).

In standard H_∞ framework, the matrices L and M of the system (10) should be determined in such a way to satisfy the following constraints

$$\begin{cases} \lim\limits_{t \to +\infty} r_e(t) = 0 & \text{if } f(t) = 0 \\ \|r_e(t)\|_2 < \gamma \|f(t)\|_2 & \text{if } f(t) \neq 0 \end{cases} \tag{11}$$

In the presence of the fault $f(t)$, the sensitivity of $r(t)$ with respect to $f(t)$ is better when the positive real γ is as small as possible. By using the Bounded Real Lemma, the problem of determining the matrices L and M is expressed as an optimization problem given by

$$\min_{P,K,M} \bar{\gamma}$$

s.t.

$$\begin{pmatrix} A^T P + PA - C^T K^T - KC & PE & C^T M^T \\ E^T P & -\gamma & -1 \\ MC & -1 & -\gamma \end{pmatrix} < 0 \tag{12}$$

where $P = P^T > 0$. After solving the optimization problem, the matrices of the residual generator are obtained by $L = P^{-1}K$ and M is obtained directly. The attenuation level is given by γ.

By analyzing the LMI (12), it appears that under the observability condition, the LMI can admit a solution. Then, if the LMI is negative definite, we have

$$\begin{pmatrix} -\gamma & -1 \\ -1 & -\gamma \end{pmatrix} < 0 \tag{13}$$