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MPC-based optimal path following for underactuated vessels

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Abstract: In this paper we propose a control law for an underactuated vessel to follow a straight line path using a line-of-sight (LOS) guidance law. To achieve optimal performance of the closed-loop system, a parameter of the LOS guidance law, the lookahead distance, is chosen to be time-varying and updated with a model predictive control (MPC) algorithm. For the closed-loop system we prove guaranteed global asymptotic stability (convergence to the path) and demonstrate in simulations that the performance of the system (fast convergence to the path with minimal overshoot) is improved compared to what can be achieved with a constant (small or large) lookahead distance. It is shown that global asymptotic stability of the closed-loop system is preserved even in the case when the MPC solver cannot find a solution to the optimization problem within given computation time limits.

Keywords: Path following, underactuated vessels, MPC, stability, nonlinear systems

1. INTRODUCTION

Path following for marine vehicles is an important practical problem. For a single vessel this problem has been investigated in a number of publications. In (Fossen *et al.*, 2003; Breivik and Fossen, 2004; Fredriksen and Pettersen, 2006) the problem has been considered for underactuated surface vessels and the proposed controllers are validated in simulations and experiments. Encarnação and Pascoal (2000) and Børhaug and Pettersen (2005) have studied the 3D case for underactuated underwater vehicles. In (Børhaug *et al.*, 2006*a*; Pavlov *et al.*, 2007) the straight line path following problem has been investigated as a part of a more complex formation control problem.

In the underactuated case, i.e., when the vehicle has less actuators than degrees of freedom, the problem of path following becomes especially challenging and requires advanced analysis to guarantee stability of the nonlinear closed-loop system. With such complications already at the stage of stability analysis of these nonlinear systems, little has been done in optimizing their performance. By optimal performance we understand fast convergence to the path, small (if any) overshoot and low sensitivity to external disturbances like ocean currents and waves. For general nonlinear systems performance optimization is poorly investigated, while practical demand for such an optimization is significant.

In this paper, which is an extension of the initial work carried out in (Nordahl, 2008), we approach the problem of performance optimization within the problem of path following for underactuated vessels. To keep the paper focused, we aim at minimizing convergence time to the path and simultaneously reducing possible overshoot, while preserving closed-loop stability. This problem is approached in the following sequence. Firstly, we choose a controller based on a line-of-sight (LOS) guidance law (Papoulias, 1991), which has a clear physical meaning. It controls the orientation of the vessel to aim at a point lying $\Delta > 0$ meters ahead of the vessel projection onto the path, see Figure 1. Parameter Δ is usually called a lookahead distance. It is known that for constant Δ larger than a certain threshold, which depends on the vessel parameters, such a controller guarantees global asymptotic and local exponential convergence to the path, see, e.g., (Pettersen and Lefeber, 2001; Fredriksen and Pettersen, 2006; Pavlov *et al.*, 2007). Small Δ corresponds to fast convergence to the path, but with a large overshoot. At the same time, large Δ reduces overshoot and results in smooth, but slow convergence. This behavior is natural since locally Δ corresponds to the inverse proportional gain (Breivik and Fossen, 2008). Introducing a time-varying $\Delta(t)$ could combine the benefits of both small and large Δ to improve the overall performance of the closed-loop system.

In this paper we will optimize $\Delta(t)$ to achieve faster convergence and smaller overshoot than what can be achieved with a constant lookahead distance as is conventionally used in LOS algorithms (Healey, 2006). The optimization of $\Delta(t)$ will be based on a model predictive control (MPC) algorithm. The idea of MPC, see, e.g., (Maciejowski, 2002), is to predict the response of a system to inputs and use these predictions to find an input which results in an optimal behavior of the system (corresponding to the minimum of some cost function) satisfying certain constraints.

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In this paper the cost function will be chosen with its minimum corresponding to (fast) exponential convergence of the vessel to the path without overshoot. The constraints in an MPC algorithm specify acceptable inputs. In our case we will consider an input as acceptable if it leads to globally asymptotically stable dynamics of the closed-loop system. This requirement will be expressed as constraints on Δ and $\dot{\Delta}$. An MPC controller taking into account these constraints provides globally asymptotically stable closed-loop dynamics with additional optimization of the transient performance. Derivation of the constraints on Δ and $\dot{\Delta}$ that lead to stable closed-loop behavior is an important contribution of this paper since such a result enables not only the use of MPC, but also of other methods for optimization of Δ within the constraints.

There exist several publications using MPC algorithms for marine control purposes. An optimal control law for cross-track control of an underactuated vessel is presented in (Børhaug *et al.*, 2006*b*). In (Marafioti *et al.*, 2008) an MPC-based controller is used for keeping an underwater vehicle a constant distance from the ocean bottom. Another MPC scheme designed for an autonomous underwater vehicle is presented in (Naeem, 2002), where a simple line-of-sight guidance scheme is utilized to generate a reference heading, which is tracked by a model predictive controller. It is common to many MPC applications that proving stability of the closed-loop system is a very difficult task. The approach adopted in this paper avoids this problem since the designed MPC controller automatically makes the closed-loop system globally asymptotically stable.

Simulations of the proposed MPC-based control scheme show that the closed-loop dynamics have better performance in terms of fast convergence to the path and minimal overshoot than for the LOS-based controllers with constant Δ. Since the guaranteed stability of the closed-loop system is not only globally asymptotical, but also locally exponential, the closed-loop system possesses a certain degree of robustness with respect to external disturbances and model uncertainties. Finally, as follows from the simulations, the computational time for this MPC controller is sufficiently low to allow for its online implementation.

The paper is organized as follows. In Section 2 the vessel model and control objective are specified. In Section 3 an LOS-based controller is presented and constraints on $\Delta(t)$ and $\dot{\Delta}(t)$ that lead to global asymptotic stability of the closed-loop system are derived. An MPC-based optimization of $\Delta(t)$ is presented in Section 4. Simulation results are provided in Section 5, while conclusions are given in Section 6.

2. VESSEL MODEL AND CONTROL OBJECTIVE

The ship model considered in this paper is given by (Fossen, 2002)

$$\dot{x} = u\cos\psi - v\sin\psi \tag{1a}$$

$$\dot{y} = u\sin\psi + v\cos\psi \tag{1b}$$

$$\dot{\psi} = r, \tag{1c}$$

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}\boldsymbol{\nu} = \mathbf{B}\boldsymbol{u} \tag{2}$$

where x and y represent the ship position and ψ the orientation relative to an Earth-fixed reference frame,

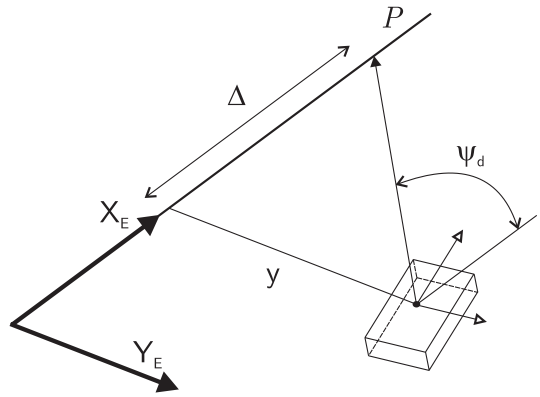

Fig. 1. Illustration of the LOS angle ψ_d

while $\boldsymbol{\nu} = [u\ v\ r]^T$ denotes the surge, sway, and yaw velocities in the body-fixed frame. The mass matrix is denoted by $\boldsymbol{M}$, $\boldsymbol{C}(\boldsymbol{\nu})$ is the Coriolis and centripetal matrix and $\boldsymbol{D}$ is the damping matrix. The vector $\boldsymbol{u} = [T_u\ T_r]^T$ is the control input corresponding to surge thrust and rudder deflection; the matrix $\boldsymbol{B}$ is a 3×2 actuator configuration matrix of the form $\boldsymbol{B} = [1\ 0; 0\ Y_\delta; 0\ N_\delta]$, for some constant parameters Y_δ and N_δ. Note that we consider underactuated vessels since only 2 independent controls are available to control 3 degrees of freedom. The origin of the earth-fixed coordinate system is placed with its x-axis aligned with the straight line path to be followed. Thus the desired motion of the vessel along the path corresponds to $y = 0$ (distance to the path) and $\psi = 0$ (vessel orientation parallel to the path). Therefore the control objective is to guarantee $y(t) \to 0$, $\psi(t) \to 0$ with the additional requirement $u(t) \to u_d$ as $t \to +\infty$, where $u_d > 0$ is a desirable constant speed along the path. Convergence to the path should preferably be fast with small or no overshoot.

3. LOS-BASED CONTROLLER AND STABILITY ANALYSIS

To solve the path following problem, we adopt the line-of-sight-based guidance, see, e.g., (Papoulias, 1991; Fredriksen and Pettersen, 2006; Pavlov *et al.*, 2007). To introduce the controller we firstly transform our model. After multiplying both sides of (2) from the left by $\mathbf{M}^{-1}$, we obtain the following model of the vessel dynamics

$$\dot{u} = F_u(u, v, r) + \tau_u, \tag{3a}$$

$$\dot{v} = Y(u)v + X(u)r, \tag{3b}$$

$$\dot{r} = F_r(u, v, r) + \tau_r, \tag{3c}$$

where τ_u and τ_r are new control inputs satisfying $\boldsymbol{M}^{-1}\boldsymbol{B}\boldsymbol{u} = [\tau_u, 0, \tau_r]^T$. Note that we assume the control input $\boldsymbol{u}$ not affecting the sway motion. As shown by Fredriksen and Pettersen (2006), this is not a restrictive assumption since for a large class of surface vessels this can be achieved by a proper choice of the body-fixed coordinate system. For details on the choice of the body-fixed coordinate system and for the expressions of $F_u(u, v, r)$, $X(u)$, $Y(u)$ and $F_r(u, v, r)$, see (Fredriksen and Pettersen, 2006).

The controller consists of two components. The first component is responsible for surge speed control:

$$\tau_u := -F_u(u, v, r) - k_u(u - u_d), \tag{4}$$

where $k_u > 0$ is a control gain. This controller is based on feedback linearization and guarantees exponential con-