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Event-Triggered Control of Sampled-Data Nonlinear Systems ⋆

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Abstract: The paper combines authors' previous results on extending Emilia Fridman's method to a class of nonlinear systems with sector bounded nonlinearity with the recent results of A.Selivanov and E.Fridman on a switching approach to event-triggered control. In this paper the sampled-data control under continuous event-trigger is considered. The closed-loop system is represented as the system switching between periodic and event-triggered sampling. Applying Fridman's method and Yakubovich's S-procedure the problem is reduced to feasibility analysis of linear matrix inequalities. Particularly it is demonstrated that the event-trigger can reduce the network workload by the example on synchronization of Chua's circuits.

Keywords: nonlinear control systems, event-trigger, sampled-data control, LMI, S-procedure

1. INTRODUCTION

Digital controllers have become a standard mean for implementation of control systems. Including the computer into the control loop leads naturally to the fact that the resulting system will have a hybrid discrete-continuous dynamics. Sampling effects have been studied intensively in the early stages of research and design of hybrid systems. Later the problem has lost its importance with the growth of computing power. However, nowadays there is a new wave of interest in the study of sampling effects due to active use of communication networks (including the Internet) in control. Due to the fact that these networks are significantly loaded, its use imposes information limitations, and an important problem of the proper choice of the sampling interval providing stability and the desired performance of the control system becomes essential. This problem is by no means trivial even for linear systems if one needs to evaluate nonconservative bounds for maximum admitted value of sampling interval. As for nonlinear systems the problem is not well studied despite its importance.

Recently in the literature an interest has grown up in a novel approach to the sampling time evaluation based on transformation of discrete-continuous system to continuous delayed system with time-varying delay. The origin of the idea can be traced back to Mikheev et al. (1988); Fridman (1992). However being combined with the descriptor method of delayed systems analysis proposed by Fridman (2001) the idea has become equipped with powerful calculation tools based on LMI and has become a powerful design method allowing one to dramatically reduce conservativity of the design Fridman et al. (2004); Fridman (2010).

In the authors' previous papers Seifullaev and Fradkov (2013, 2016) the extension of Fridman's method to a class of nonlinear systems with sector bounded nonlinearities was proposed. The aim was to evaluate the upper bound of the sampling interval below which the system is absolutely stable. The effect of sampling was considered as delay followed by the construction and use of Lyapunov–Krasovskii functional Fridman (2010). With S-procedure Yakubovich et al. (2004) the estimation of sampling step was reduced to feasibility analysis of linear matrix inequalities.

To reduce the amount of information transmitted over network one can use continuous event-trigger that allows to avoid the packets when the output signal does not significantly change the control signal. In Selivanov and Fridman (2015) a switching approach to event-triggered control was proposed, where the closed-loop system was represented as switching system between periodic sampling and event-trigger. In the current paper we demonstrate the extension of this approach to a class of nonlinear systems with sector bounded nonlinearity. Two examples (one of them is the problem of Chua's circuit synchronization), confirming the possibility of reducing the workload of the network with this approach, are given in Section 5.

2. PROBLEM STATEMENT

Consider the nonlinear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + q\xi(t) + Bu(t), \\ y(t) &= C^{\mathrm{T}}x(t), \quad \sigma(t) = r^{\mathrm{T}}x(t), \\ \xi(t) &= \varphi(\sigma(t), t), \end{aligned} \tag{1}$$

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where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^l$ and $\sigma(t) \in \mathbb{R}$ are the outputs, $u(t) \in \mathbb{R}^m$ is the control input, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{n\times l}$ are constant matrices, $q \in \mathbb{R}^n, r \in \mathbb{R}^n$ are constant vectors.

Assume that $\xi(t) = \varphi(\sigma(t), t)$ is the nonlinear function (see Fig.1) satisfying sector condition

$$\mu_1 \sigma^2 \leqslant \sigma \xi \leqslant \mu_2 \sigma^2, \tag{2}$$

for all $t \geqslant 0$ where $\mu_1 < \mu_2$ are real numbers.

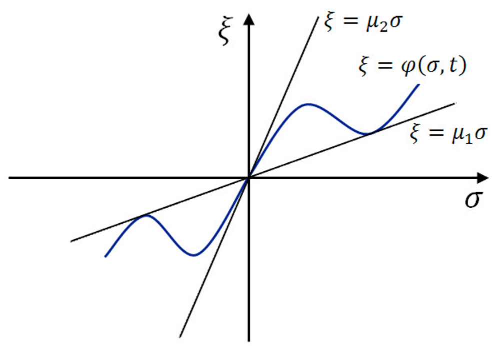

Fig. 1. Sector bounded nonlinearity

Remark 1. Let us provide some examples of sector bounded nonlinearities, satisfying (2):

- $\xi = sin(\sigma)$: $\mu_1 \approx -0.2173$, $\mu_2 = 1$ (see. Fig. 2),

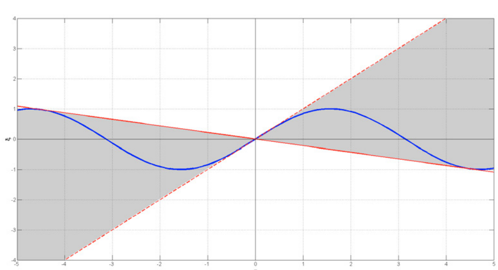

Fig. 2. $\xi = sin(\sigma)$

- $\xi = sin(\sigma^2)$: $\mu_1 \approx -0.855$, $\mu_2 \approx 0.855$ (see. Fig. 3),

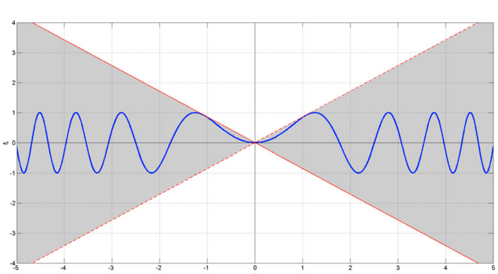

Fig. 3. $\xi = sin(\sigma^2)$

- relay with dead zone, saturation, piecewise-linear function etc. (see Khalil (2002)).

Consider a sampled-time feedback control law

$$u(t) = Ky(t_k), \ \ t_k \leqslant t < t_{k+1}, \tag{3}$$

where $K \in \mathbb{R}^{m\times l}$, and $\{t_k\}$ is an infinite sequence of sampling times $0 = t_0 < t_1 < \ldots < t_k < \ \ldots$.

Based on the input-delay approach Mikheev et al. (1988); Fridman et al. (2004) the closed loop system

$$\dot{x}(t) = Ax(t) + q\xi(t) + BKC^{\mathrm{T}}x(t_k) \tag{4}$$

can be rewritten as the system with variable "sawtooth" delay:

$$\dot{x}(t) = Ax(t) + q\xi(t) + BKC^{\mathrm{T}}x(t - \tau(t)), \tag{5}$$

where $\tau(t) = t - t_k$, $t_k \leqslant t < t_{k+1}$.

Assume that the sampling time sequence is generated by continuous event-trigger

$$t_{k+1} = \min\left\{t \geqslant t_k + h \,|\, (y(t) - y(t_k))^{\mathrm{T}}\Omega\,(y(t) - y(t_k)) \geqslant \varepsilon\, y^{\mathrm{T}}(t)\,\Omega\, y(t)\right\}, \tag{6}$$

where $\Omega \in \mathbb{R}^{p\times p}$ is a constant positive semi-defined matrix, h, ε are nonnegative scalars. Since event-trigger (6) waits for h value until checking the switching condition, it can avoid Zeno phenomenon. Such event-trigger was used in, e.g. Tallapragada and Chopra (2012a,b); Selivanov and Fridman (2015).

It is required to establish the exponential stability conditions of the system (1),(3) under event-trigger (6).

3. A SWITCHING APPROACH

In Selivanov and Fridman (2015) an approach to sampling of linear systems consisting in considering the switching between periodic sampling and continuous event-trigger was proposed. According to this approach system (1), (3) can be considered as the following switching system:

$$\dot{x}(t) = \begin{cases} Ax(t) + q\xi(t) + BKC^{\mathrm{T}}x(t - \tau(t)), \\ \qquad\qquad\qquad \text{if } t \in [t_k, t_k + h), \\ (A + BKC^{\mathrm{T}})x(t) + q\xi(t) + BKe(t), \\ \qquad\qquad\qquad \text{if } t \in [t_k + h, t_{k+1}), \end{cases} \tag{7}$$

where

$$\begin{aligned} &y(t) = C^{\mathrm{T}}x(t), \quad \sigma(t) = r^{\mathrm{T}}x(t), \quad \xi(t) = \varphi(\sigma(t), t), \\ &\tau(t) = t - t_k \leqslant h, \quad t \in [t_k, t_k + h), \\ &e(t) = y(t_k) - y(t), \quad t \in [t_k + h, t_{k+1}). \end{aligned} \tag{8}$$

4. STABILITY ANALYSIS BASED ON LYAPUNOV–KRASOVSKII FUNCTIONAL METHOD AND S-PROCEDURE

Definition 1. The space of absolutely continuous on $[-h, 0)$ functions $f : [-h, 0] \to \mathbb{R}^n$ having square integrable first-order derivatives is denoted by W with the norm

$$\|f\|_W = \max_{\theta\in[-h,0]} |f(\theta)| + \left[\int_{-h}^{0} |\dot{f}(s)|^2 ds\right]^{\frac{1}{2}}.$$

Denote $x_t(\theta) : [-h, 0] \to \mathbb{R}^n$ as $x_t(\theta) = x(t + \theta)$, where $x(\theta) \equiv 0$ if $\theta \in [-h, 0)$.

Definition 2. System (7), (8) will be called exponentially stable with the decay rate $\alpha > 0$ if there exists $\beta > 1$ such that for solution $x(t)$ of (7), (8) with initial condition x_{t_0} the following estimate holds

$$|x(t)|^2 \leqslant \beta e^{-2\alpha(t-t_0)}\, \|x_{t_0}\|_W^2, \ \forall t \geqslant t_0.$$

The proof of our main result is based on the following auxiliary statement that can be proved along the lines of Lemma 1 in Fridman (2010).