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Synchronization of coupled Hindmarsh-Rose neurons: Effects of an exogenous parameter ⋆

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Abstract: Systems that show patterns or cycles are found throughout nature. The existence of interaction mechanisms among these systems may generate an overall new collective dynamic behavior such as synchronization. But not only may the interconnection among systems lead to synchronization, also the influences of the environment plays an important role in the stablishment of collective behavior. In this paper we study how synchronization of two diffusively coupled Hindmarsh-Rose neurons is affected by an exogenous parameter. In particular, we investigate by means of numerical simulations how the threshold of the coupling strength that is needed to synchronize depends on the value of the exogenous parameter. For those values of the exogenous parameter for which the overall behavior of the two coupled neurons is periodic we perform a local stability analysis of the synchronous state.

Keywords: Synchronization; periodic oscillators; Floquet Theory; external parameter.

1. INTRODUCTION

Synchronization is a persistent type of time-correlated behavior of dynamical systems. This synchronized behavior is seen in many scenarios throughout nature (e.g. bird flocks, schools of fishes (Shaw, 1978), firing of fireflies (Buck, 1988)). In mammalians synchronization arises in cases such as bursting activity in brain cells, circadian rhythms, (Enright, 1980) body temperature, pacemaker cells of the heart (Jalife, 1984), among others.

Disregarding of the scale considered, synchronized behavior emerges due to *coupling* among individual units and differs for each system. For example, a school of fish synchronizes due to a highly-specialized sensory structure. By sensing the environment they keep trace of their position within the school, and respond quickly to changes in both water currents and movement of the group (Shaw, 1978). Many other biological synchronization phenomena can be found in literature (Strogatz, 2003; Watts et al., 1998; Winfree, 1967, 1980, 1987).

Synchronization for engineering has been studied for many years (e.g. electric grids (Blaabjerg et al., 2006), robotic swarms (Vatankhah et al., 2009), among others). These applications show the importance of the understanding of this topic. Many characteristics can be studied about this phenomenon, such as, coupling strength, topology, time-delays, and the effects of exogenous parameters (external parameter) on synchronization. Whereas the properties that lead to synchronization have been studied extensively, the extent to which the conditions for synchronization depend on an exogenous parameter has received little attention. An interesting case for synchronization is a network of neurons. Their interaction is made via chemical synapses or electrical synapses. Synchrony among neurons is seen in, for instance, the suprachiasmatic nuclei (SCN). The SCN is an endogenous biological clock, which is formed by a network of approximately 20000 neurons. Being Light the main exogenous parameter that entrains the SCN (Rohling et al., 2006). It has been observed in several studies that frequency of oscillatory activity of the neurons inside the SCN is high during the day and low during the night. This variation in frequency may influence synchronization among neurons.

Little direct evidence can be found in literature for synchronization and entrainment induced by an external parameter. The available evidence is related to neuroscience; for example Gonze et al. (2005) studied coupling mechanisms through global level of neurotransmitter concentration cells entrained by 24hr light-dark cycles. Another example of network of neurons is presented in the work of Kunz et al. (2003). They show simulations of a large network of coupled Van der Pol oscillators. The systems are arranged in a 2-D lattice, and they included an external parameter called *perceived brightness*, defined by certain protocols of light-dark cycles. Their simulations showed clusters and robustness against noise. Moreover, Will (2007) argued that not only light stimuli influence synchronization in the brain, but also external periodic

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acoustic stimuli influences synchronization and entrainment.

In this work, we study the effects of an exogenous parameter (or bifurcation parameter) on a network. We analyze the synchronization properties of two coupled neurons affected by changing the bifurcation parameter in each system by the same amount and showing the effects of this variation on the synchronization threshold.

We present simulations of two coupled oscillators. Each has Hindmarsh-Rose (H-R) model dynamics, and the coupling is diffusive (i.e. the weighted difference of the output of the neurons). It is known that, for a sufficiently strong coupling, the oscillators synchronize (Steur et al., 2009). We present the effects of an external parameter affecting the behavior of each neuron, the repercussion to the overall synchronization. Simulations are performed to find a threshold of coupling strength for synchronization as function of the external parameter. Depending on the bifurcation parameter, the H-R neurons exhibit different behaviors, such as resting, oscillatory and chaotic. In the case that the synchronized solutions are periodic, Floquet theory is used to perform a local analysis and verify stability in the periodic region of the neurons.

This paper is organized as follows: In Section 2, we present the single-oscillator and multi-oscillator models of Hindmarsh-Rose equations. Also, the different behaviors of a neuron originated by the bifurcation parameter. Section 3 describes the results of simulations and the coupling strength threshold of synchronization. Section 4 discusses a Floquet analysis for the periodic regimes. Concluding remarks are presented in Section 5.

2. METHODS

Several models of neuron oscillators can be found in literature, for example Hodgkin-Huxley, FitzHugh-Nagumo, Hindmarsh-Rose, among others. In this work, we use the Hindmarsh-Rose (H-R) model, which provides one of the simplest models of the more general phenomenon of oscillatory (bursting) discharge (Hindmarsh et al., 1984).

2.1 Single-oscillator model

The dynamic equations of the H-R neuron are defined as

$$\begin{aligned} \dot{z}_1 &= 1 - 5y^2 - z_1 \\ \dot{z}_2 &= 0.005(4(y + 1.6180) - z_2) \\ \dot{y} &= -y^3 + 3y^2 + z_1 - z_2 + I \end{aligned} \qquad (1)$$

where y is the output potential of the neuron, z_1 and z_2 are internal variables, and I is a bifurcation parameter (applied current), which regulates the behavior of the neuron.

Changing the applied current yields a change of the dynamics of the model. For $0 \lesssim I \lesssim 1.4$ the neuron exhibits a *Resting state*, which means that the states converge to a stable equilibrium point.

When increasing the bifurcation parameter, the states yield to limit cycles with different behavior. For instance, a *bursting* mode is present for values between $1.4 \lesssim I \lesssim$ 3.1, this behavior is characterized by periods of firing that alternate with periods of quiescence. An example is shown in the top of Figure 1, where, for a bifurcation parameter $I = 2$, the output shows a periodic bursting with three spikes per burst. For a range $3.1 \lesssim I \lesssim 3.5$ *chaotic* behavior occurs. an example of chaotic bursting is shown in the middle panel of Figure 1. This means that no fixed number of bursting spikes exists. In other words, for infinite time the trajectory of the model never converges to a unique (periodic) solution, and the model exhibits some long term aperiodic behavior. In addition, the behavior depends sensitively on initial conditions. When the input current takes values of $I \gtrsim 3.5$ the model exhibits *tonic spiking* behavior, see the bottom panel of Figure 1. Frequency of oscillations increases as of I increases.

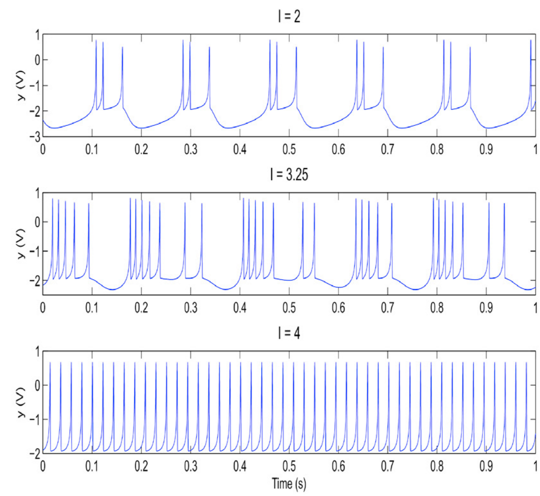

Fig. 1. Different firing behaviors of a H-R model: Bursting ($I = 2$), Chaotic ($I = 3.25$), Tonic Spiking ($I = 4$)

A bifurcation diagram that illustrates the behavior of the H-R model is depicted in Figure 2. It shows the boxplots of the average number of spikes fired per time unit for different values of bifurcation parameter. In the figure we observe the various modes mentioned above. Periodic bursting is shown as a light grey area, having a persistent average number of spikes. White area is the chaotic mode, also seen as the zoom-in box, exhibits a relatively large standard deviation on the average number of spikes per time unit. Dark grey region represents the tonic spiking region, showing that the frequency of spikes increases proportionally to the bifurcation parameter. Due to the different initial conditions taken on each simulation, a small non-zero variance is visible in the average number of spikes on light and dark grey regions.

2.2 Synchronization in a multi-oscillator model

As being mentioned in the introduction, there are two types of synapses that establish interaction between neurons: chemical synapses and electrical synapses. The latter is a direct electrical link between neighboring neurons. These electrical synapses form a narrow gap between pre- and postsynaptic neurons are also known as gap junctions, (Kandel et al., 2012). An approximation to this interaction