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Hybrid Trajectory Tracking for a Hopping Robotic Leg

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Abstract:
Reference-spreading based trajectory tracking is a recently introduced control strategy for hybrid systems with state-triggered jumps that allows to handle the time mismatch between nominal and closed-loop impact times. In this paper, we demonstrate that the approach can handle even the change in state dimension corresponding to, e.g., the activation of a unilateral contact constraint occurring when dealing with mechanical systems experiencing inelastic impacts. We demonstrate the effectiveness of the approach by means of simulations, addressing a trajectory tracking problem for a two-link robotic leg performing a hopping motion. The robot leg is modeled as a hybrid system possessing two different dynamic phases (stance and flight) with different state dimensions. Robustness of the approach, in particular to inexact measurement of robot height with respect to the ground, is also discussed.

Keywords: Hybrid systems, trajectory tracking, hybrid control, robot leg, locomotion

1. INTRODUCTION

In performing locomotion and manipulation tasks, a robot has to make and brake contact with the environment several times. Designing control strategies to, e.g., make a humanoid robot walk, jump, or climb are extremely challenging due to the hybrid nature of the dynamics, which switches between discrete modes based on the number of active contacts established with the environment.

This paper wants to demonstrate that a recently introduced hybrid trajectory tracking controller Saccon et al. (2014); Rijnen et al. (2015) might be an interesting control strategy to achieve stable transition between the various discrete modes naturally occurring when a robot is performing contact tasks.

The contribution is twofold. First, we extend the approach presented in Saccon et al. (2014); Rijnen et al. (2015) to the situation where the hybrid system is made of multiple dynamic modes with different dimension of the corresponding state. In Saccon et al. (2014), a novel notion of tracking error is introduced by extending the reference trajectory in a neighborhood of the nominal contact times, obtaining so-called extended ante- and post-event reference trajectories. While in Saccon et al. (2014) the analysis is limited to a single event, in Rijnen et al. (2015) the theory is fitted to the framework of hybrid time (Goebel et al., 2012, chapter 2) to account for multiple impact events, use the jumping linearization in an LQR-like design of the tracking gains, and to better connect it with literature on dynamics and control of hybrid systems. However, both Saccon et al. (2014) and Rijnen et al. (2015) limit the analysis to the case of (partially) elastic impacts, thus keeping the (effective) dimension of the state space before and after the impact the same.

Secondly, we demonstrate the proposed theory in simulation using a dynamic model of the robotic leg described in Tsagarakis et al. (2013). This work is part of an ongoing collaboration between TU/e and IIT (previous working location of the third author) to demonstrate stable hopping on the existing robot setup. A hopping motion can be divided into two phases: a first phase where the leg floats in mid-air (flight) and a second phase where the leg is in contact with the ground (stance). Upon transition from flight to stance phase (touch-down), a sudden modification of velocity is generally witnessed. The very small time scale of such impact event (compared to the time span of stance and flight phases) motivates the modeling assumption that the velocity variation is due to an impulsive force and that it occurs, therefore, in zero time. The resulting dynamics is therefore hybrid, exhibiting both continuous (flow) and discrete (jump) dynamics Goebel et al. (2012). The theoretical framework of hybrid systems with state-triggered jumps is suitable to describe dynamical models in the area of robotics as well as rigid body mechanics with unilateral contact constraints as mentioned in Ding et al. (2011).

Trajectory tracking for hybrid systems with state-triggered jumps such as the jumping robot leg discussed in this paper is complicated due to the hybrid nature of the system dynamics. The literature on tracking control of such a class of hybrid systems for a given time-varying reference

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trajectory is relatively limited and it represents an active field of research. Recent (theoretical and experimental) techniques are provided in Pagilla and Yu (2001); Menini and Tornambè (2001); Leine and van de Wouw (2008); Forni et al. (2013); Biemond et al. (2013); Saccon et al. (2014); Incremona et al. (2015). The main difficulty is finding a proper notion of error as the jump times of plant and reference trajectory cannot be assumed to coincide. This point of view is also taken in this paper.

The control of a hopping robot has been considered by many researchers in the field. One of the fundamental contributions is presented in Raibert (1984), based on energy consideration. A lot of different robot designs and methods to control them have been considered in literature, see for example Lebaudy et al. (1993); Michalska et al. (1996); Gregorio et al. (1997); Mathis and Mukherjee (2013).

Going beyond hopping, methods to control robot locomotion, particularly bipedal walking, include the use of virtual (non-)holonomic constraints, Poincaré maps, and transverse linearization such as described in, e.g., Hera et al. (2013) and Grizzle et al. (2014). Other approaches to stability analysis and control of rigid-body systems with impacts and friction are presented in Posa et al. (2015) and, more specifically, for planning and control of nonperiodic bipedal locomotion, in Zhao et al. (2015).

This paper is organized as follows. In Section 2, we discuss the hybrid feedback control law as introduced in Saccon et al. (2014); Rijnen et al. (2015) and adapt the notation to handle multiple modes. In Section 3, this control strategy is applied to the robotic leg introduced in Tsagarakis et al. (2013) in performing a hopping motion. A model of the setup is presented first in Section 3.1, followed by trajectory tracking results (in simulation) in Section 3.2. The conclusions of this work are presented in Section 4.

2. HYBRID CONTROL

As a starting point, we consider the trajectory tracking control law for hybrid systems with state-triggered jumps first introduced in Saccon et al. (2014). The control strategy uses so-called extended reference trajectories to form a novel definition of error that is still valid even when the hybrid system is composed of different *modes* with different state dimension. From here on, this control strategy will be referred to as *hybrid trajectory tracking control based on reference spreading* or *RS-based hybrid control* for short. In this section, the control law will be explained. We will use the notation in Rijnen et al. (2015) and adapt it to handle multiple mode systems.

Consider Fig. 1. The figure is used to provide, at a glance, the notation that we will use in this paper to detail the dynamics of a hybrid system. The system consists of three distinct modes (gray circles), with corresponding discrete time state $\sigma \in \{1, 2, 3\}$. The continuous time state $x \in \mathbb{R}^{n_\sigma}$ of the system changes both during continuous evolution within a mode (flow) and whenever a transition between modes occurs (jump). To describe this combined evolution, we consider the notion of hybrid time (see Goebel et al. (2012)) where the continuous time $t \in [t_0, t_f]$ is combined with an event counter $j \in \{0, 1, 2, \dots\} = \mathbb{N}$ to form the hybrid time instant (t, j). In this, t_0 is the initial time and t_f the final time. The continuous time state x of the system at time t when in mode σ evolves along the vector field

$$\dot{x} = f^\sigma(x, u, t, j), \tag{1}$$

with $u \in \mathbb{R}^{m_\sigma}$ the input. A transition from an ante-event mode σ_a to post-event mode σ_p is enabled when $\gamma^{\sigma_p \leftarrow \sigma_a}(x, t, j, u) = 0$, with $\gamma^{\sigma_p \leftarrow \sigma_a} : \mathbb{R}^{n_{\sigma_a}} \times \mathbb{R} \times \mathbb{N} \times \mathbb{R}^{m_{\sigma_a}} \to \mathbb{R}$ a smooth real-valued function referred to as a *guard function*. In the case of such a transition, the state is reset according to

$$x(t, j+1) = g^{\sigma_p \leftarrow \sigma_a}(x(t, j), t, j), \tag{2}$$

with $g : \mathbb{R}^{n_{\sigma_a}} \times \mathbb{R} \times \mathbb{N} \to \mathbb{R}^{n_{\sigma_p}}$, $(x, t, j) \mapsto g(x, t, j)$ referred to as the *jump map*. An enabled transition from and to the same mode is referred to as a self-loop.We only allow the evolution in (1) to continue if all guard functions corresponding to that particular mode satisfy $\gamma^{(\cdot)\leftarrow\sigma} > 0$.

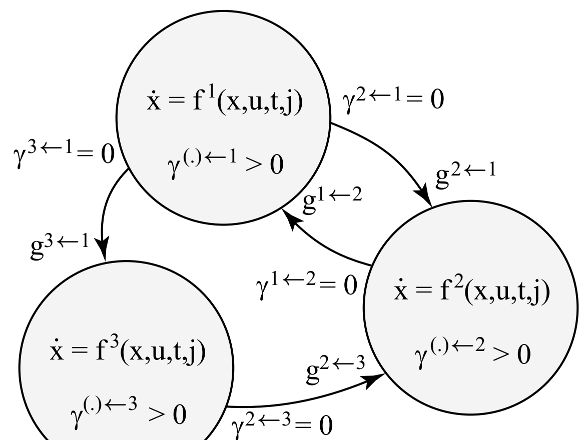

Fig. 1. Finite state machine representation of a hybrid system with 3 distinct modes (circles) and the corresponding enabled mode transitions (arrows).

For a trajectory x that is a solution to (1)-(2) for initial condition $x(t_0, 0) = x_0$, initial mode σ_0, and input signal $u(t, j)$, we denote the time t of the i-th event by t_i, with $i \in \{1, 2, \dots, N\}$, where N is the total number of events, possibly infinite. The hybrid time domain of this trajectory is defined as

$$\text{dom } x := \bigcup_{j=0}^{N} \left(I_x^j \times \{j\} \right), \tag{3}$$

with $I_x^j := [t_j, t_{j+1}]$ the closed time interval between the j-th and $(j+1)$-th event. See Fig. 2 for examples of hybrid time domains (namely dom x and dom α). The event times of $x(t, j)$ are captured in the set

$$E_x := \bigcup_{j=1}^{N} \left(\{t_j\} \times \{j-1\} \right). \tag{4}$$

Tracking (feedback) control is based on suitably choosing the input u so as to steer the system toward a desired time-varying reference signal. To make a clear distinction between the state of the tracking system and that of the reference trajectory we denote the reference by $\alpha(t, j)$ which is the solution to (1)-(2) for input $u = \mu(t)$ and initial condition $\alpha(t_0, 0) = \alpha_0$. For the reference, ς denotes the mode, with initial condition ς_0. Furthermore, the j-th event time of the reference trajectory is denoted τ_j,