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Standardizability of linear periodic sampled-data systems ⋆

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Abstract: Due to periodic sampling, linear sampled-data system are a subclass of linear continuous periodic systems, even in the case, when all other elements are time invariant. Owing to the great practical importance of sampled-data control systems, various approaches for the rigorous description of those systems are known. Moreover, a lot of methods have been developed, that are able to yield rigorous solutions for numerous control and optimization problems. The application of those methods need representations in certain standard forms. Often, it is not clear, whether a given system can be transformed into a standard form or not. The paper considers this question for the standard sampled-data system, for which numerous methods and tools are available, if the system belongs to the subclass with model structure. The paper provides necessary and sufficient conditions for a SD system with standard structure to belong to this important subclass. At hand of a simple SD system, it is shown that the standard structure does not imply model standardizability.

Keywords: Linear systems, Periodic sampling, Sampled-data systems, Transfer functions, MIMO systems

1. INTRODUCTION

Sampled-data (SD) systems are characterized by the presence of as well continuous as discrete-time elements. Due to the massive introduction of digital controllers and filters, SD systems are of great practical importance. In case of linear time-invariant processes and controllers, there exist two simple approaches for analysis and design of SD systems. The first one is called quasi-continuous approach. Here the system is considered as purely continuous system, and e.g. the controller is design as a continuous one. After that the continuous controller is substituted by a digital approximation, e.g. by fast sampling and Tustin's formula. The second approach is a pure discrete one. Here the not necessarily fast sampled process is substituted by a discrete model. Then analysis and design are completely done in discrete time, e.g. by using z-transforms. However, in this case no information about the intersample behavior is provided. Detailed descriptions of both approaches can be found in standard textbooks, e.g. Åström and Wittenmark [1997], Franklin et al. [2002]. Nevertheless, both approaches are approximations, and already in simple cases the results can be unusable, Rosenwasser and Lampe [2000], Yamamoto et al. [2002].

Therefore, since the beginning of the 1990th, new methods with exact and complete information have been developed for SD systems. This methods are referred to, when the concept of sampled-data is used in a stricter sense. However, due to the periodic sampling, a SD system is a continuous periodic system, and the standard methods for LTI systems are no longer applicable, Kabamba and Hara [1993]. Three approaches have been prepared for analysis and design of SD control systems. The lifting technique is based on state space representations and it implies a transfer to representations with infinite input and output spaces, Bamieh and Pearson [1992], Yamamoto [1994]. Various control and optimization problems for SD systems have been solved by lifting, e.g. Chen and Francis [1995] and the references in it. A corresponding approach in the frequency domain is called FR operator, Hagiwara and Araki [1995], which also leads to operations with infinite dimensional matrices, in particular the Hill matrix occurs, Hill [1886].

The parametric transfer function (PTF) concept as the third exact method is applied in the present paper. The PTF is a generalization of the well-known ordinary transfer function for LTI systems to the case of linear time-varying systems. For MIMO SD systems it operates with parametric transfer matrices (PTM), but these matrices are of finite dimension. The concept has proved to solve manifold control problems for SD systems or other kind of continuous periodic systems, including stability and modal control, $\mathcal{H}_2$ and $\mathcal{L}_2$ optimization or advanced statistical analysis, Rosenwasser and Lampe [2000, 2006], Lampe and Rosenwasser [2012] and the references therein.

2. STATEMENT OF PROBLEM

1) In mathematical-technical literature, e.g. Åström and Wittenmark [1997], Chen and Francis [1995], Rosenwasser and Lampe [2006], the multiple-input multiple-output (MIMO) sampled-data (SD) system is studied, in which

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the continuous process is described in state space by the equations

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = Av(t) + B_1x(t) + B_2u(t), \qquad y(t) = C_2v(t), \tag{1}$$

where $y(t), v(t), x(t), u(t)$ are vectors of dimensions $n \times 1$, $\chi \times 1$, $\ell \times 1$, $m \times 1$, respectively, and $A,\ B_1,\ B_2,\ C_2$ are constant matrices of appropriate size. Assume that the pair (A, B_2) is controllable, and the pair (A, C_2) is observable.

2) Furthermore, assume that the discrete part is discribed by the control program (CP)

$$\alpha(\zeta)\psi_k = \beta(\zeta)y_k, \quad (k = 0, \pm 1, ...), \tag{2}$$

where ψ_k is an $m \times 1$ vector. The sample and hold element (S&H) is described by an ideal sampler and a zero-order hold with free shape of the impulses:

$$y_k = y(kT), \qquad u(t) = h(t - kT)\psi_k, \quad kT < t < (k+1)T. \tag{3}$$

In equation (2), the quantities

$$\alpha(\zeta) = \alpha_0 + \alpha_1\zeta + ... + \alpha_\rho\zeta^\rho, \quad \det\alpha_0 \neq 0, \qquad \beta(\zeta) = \beta_0 + \beta_1\zeta + ... + \beta_\rho\zeta^\rho \tag{4}$$

are polynomial matrices in the backward shift operator ζ, Åström and Wittenmark [1997], i.e.

$$\zeta\psi_k = \psi_{k-1}, \quad \zeta y_k = y_{k-1}. \tag{5}$$

Moreover, in (2) the quantities α_i and β_i are constant $m \times m$ and $m \times n$ matrices, respectively. Assume that the polynomial pair $(\alpha(\zeta), \beta(\zeta))$ is left coprime in the sense of Kailath [1980]. Finally, in (2) the quantity T is the sampling period, and $h(t)$ is a scalar function giving the shape of the impulses, and it is of bounded variation on the interval $0 \leq t \leq T$. As output of the sampled-data system, we consider the $r \times 1$ vector

$$z(t) = C_1v(t) + Du(t), \tag{6}$$

where C_1, D are constant matrices of size $r \times \chi$ and $r \times m$, respectively.

All together, equations (1) - (6) describe a certain linear periodically nonstationary continuous-discrete system. Below, the system of differential-difference equations (1) - (3), (6) is called standard model of a sampled-data system and it is denoted by $\mathcal{S}_T$.

3) As was derived in Rosenwasser and Lampe [2006], by applying the Laplace transformation, the system $\mathcal{S}_T$ can be configured to the general structure shown in Fig. 1, which is called standard structure.

In Fig. 1, the blocks are determined by

$$K(s) = C_1(sI_\chi - A)^{-1}B_1, \qquad L(s) = C_1(sI_\chi - A)^{-1}B_2 + D, \qquad M(s) = C_2(sI_\chi - A)^{-1}B_1, \qquad N(s) = C_2(sI_\chi - A)^{-1}B_2 \tag{7}$$

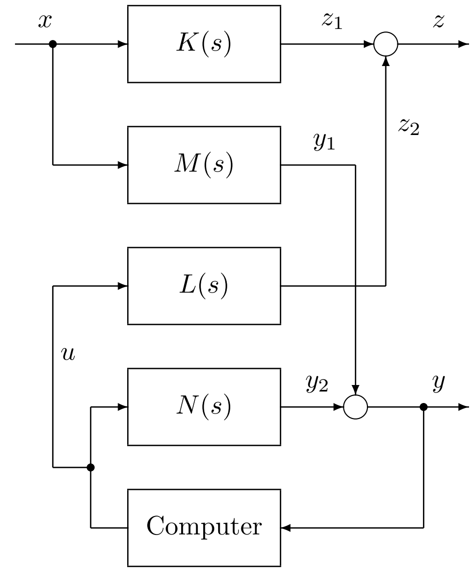

Fig. 1. Standard structure of sampled-data system

which are rational matrices in the complex variable s. The computer block contains CP (2) with S&H (3), and I_χ denotes the $\chi \times \chi$ identity matrix.

4) As was derived in Rosenwasser and Lampe [2006], if $x(t) = I_\ell\,\mathrm{e}^{\lambda t}$, where λ is a complex parameter, then for all λ, excluding a certain countable set of values, there exist a unique matrix solution of equations (1) - (3), (6), where

$$\begin{aligned} y(\lambda,t) &= \mathrm{e}^{\lambda t}\,W_{yx}(\lambda,t), \quad W_{yx}(\lambda,t) = W_{yx}(\lambda,t+T), \\ v(\lambda,t) &= \mathrm{e}^{\lambda t}\,W_{vx}(\lambda,t), \quad W_{vx}(\lambda,t) = W_{vx}(\lambda,t+T), \\ u(\lambda,t) &= \mathrm{e}^{\lambda t}\,W_{ux}(\lambda,t), \quad W_{ux}(\lambda,t) = W_{ux}(\lambda,t+T), \\ z(\lambda,t) &= \mathrm{e}^{\lambda t}\,W_{zx}(\lambda,t), \quad W_{zx}(\lambda,t) = W_{zx}(\lambda,t+T), \end{aligned} \tag{8}$$

and, moreover,

$$\psi_k(\lambda) = \mathrm{e}^{\lambda T}\,\psi_{k-1}(\lambda). \tag{9}$$

In equations (8), (9) the matrices $W_{yx}(\lambda,t)$, $W_{vx}(\lambda,t)$, $W_{ux}(\lambda,t)$, $W_{zx}(\lambda,t)$ and $\psi_k(\lambda)$ have the dimensions $n \times \ell$, $\chi \times \ell$, $m \times \ell$, $r \times \ell$ and $m \times \ell$, respectively. Below, in analogy to Rosenwasser and Lampe [2006], the matrices $W_{yx}(\lambda,t)$, $W_{vx}(\lambda,t)$, $W_{ux}(\lambda,t)$, $W_{zx}(\lambda,t)$ are called parametric transfer matrices (PTM) from the input $x(t)$ to the corresponding outputs y, v, u, z.

5) The in Rosenwasser and Lampe [2006] obtained expression for the PTM $W_{zx}(s,t)$ has the form

$$W_{zx}(s,t) = \phi_{L\mu}(T,s,t)\tilde{R}_N(s)M(s) + K(s). \tag{10}$$

Herein