Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-3 (2016) 165–170

# Finite time tracking control of higher order nonlinear multi agent systems with actuator saturation

**Sanjoy Mondal** * **Rong Su** **

* *Air Traffic Management Research Institute, Nanyang Technological University, Singapore-639798 (e-mail: mondals@ntu.edu.sg).*
** *School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore-639798 (e-mail: rsu@ntu.edu.sg)*

**Abstract:** In this paper a distributed leader follower finite time tracking control of higher order nonlinear multi agent systems (MAS) is presented subject to actuator saturation. A saturated continuous homogeneous consensus control is developed to obtain finite time convergence. For stability of the saturated actuators the geometric homogeneity theory is used and it is proven that all the states of the followers can converge to that of the leader in finite time. Switching control is designed based on super twisting algorithm to nullify the effect of uncertainties and external disturbances. It is also ensures that the sliding surface reaches the equilibrium in finite time. The control law can be effectively used for more general higher order nonlinear agent dynamics. Simulation results verify the effectiveness of the proposed scheme.



*Keywords:* Multi agent systems, finite time control, actuator saturation.

## 1. INTRODUCTION

In cooperative control a group of agent reaches a common agreement based on their local information exchange. In recent years consensus problem has found many transportation and engineering applications such as coordination control of marine vehicles Ren and Beard (2008), consensus control of quadrotors for transportation, air traffic control Weigang et al. (2008), cooperative control of unmanned air vehicles Pack et al. (2009); Ghommam and Saad (2014), autonomous underwater vehicles Yoon and Qiao (2011) and so on. Among many existing traditional consensus control approaches Ghasemi et al. (2014); Khoo et al. (2009); Zhao et al. (2011), the distributed control approach for multi-agent systems (MAS) provides many advantages, such as stronger robustness gainst the uncertainties, less communication requirements and higher efficiency. In the leader-follower formation, the leader is usually independent of its followers, but can affect the follower's behaviours, while the leader's behaviour can easily be controlled, in order to achieve the control objective. For real time practical control systems actuator saturation is one of the most common existing nonlinearities since the capability of any physical actuator is limited. If the effect of actuator saturation is neglected, it may cause inferior control performance as well as it can make a system unstable. So in consensus problem the effect of actuator saturation cant not be neglected.

A leader-following consensus problem for a group of linear identical agents subject to control input saturation was studied in Meng et al. (2013). In Zhang et al. (2014) a distributed finite-time observer is designed for second order MAS, where the control inputs are required to be bounded. In Wei et al. (2014) a high gain feedback controller is designed for the tracking problem of leader follower MAS subject to actuator saturation. Which can achieve asymptotic stability. Most of the above consensus control problems are solved asymptotically with infinite setting time. However, faster finite-time convergence rate is an important indicator for the dynamic behaviors of the agents and it is often required in consensus problems also. As a consequence, finite-time control has received great interest in the control community Ghasemi and Nersesov (2014); Ghasemi et al. (2014); Khoo et al. (2009); Zuo (2015). Compared with asymptotical control or exponential control technique, finite-time control offers less control effort, faster response, higher accuracy, and better disturbance rejection and robustness against uncertainties.

Many results of finite time network consensus control using sliding mode are reported in the literature Ghasemi and Nersesov (2014); Ghasemi et al. (2014); Khoo et al. (2009). The major benefit of sliding mode control is its robustness against the uncertainties. As we know, the dynamics of lots of mechanical systems can be modeled as double-integrators. Thus the MAS with double-integrator dynamics have been paid great attention in recent years Ghasemi and Nersesov (2014); Ghasemi et al. (2014); Khoo et al. (2009). Recently, MAS with general high-dimensional linear dynamics have attracted much attention of researchers in the control field Ma and Zhang (2010). So designing consensus algorithms using sliding mode for higher order MAS is still a considerable challenge. A fixed time consensus tracking problem for homogeneous second-order MAS is developed in Zuo (2015), where the agent dynamics are chosen as a simple double integrator system only. Recently an integral sliding mode control proposed to counteract the effect of uncertainties is proposed for double integrator MAS Yu and Long (2015).


10.1016/j.ifacol.2016.07.028

With regard to the finite-time control problem using sliding mode, there are still two difficulties to be settled. First all these new consensus control techniques Ghasemi and Nersesov (2014); Ghasemi et al. (2014); Khoo et al. (2009); Zhao et al. (2011) can only be used when the agent dynamics are of the second order, i.e. double integrators only. Thus it can be very restrictive, as the agent dynamics can be of any order. Second, in the consensus control of MAS, with the interaction information between all its neighbors each agent updates its control protocol. However, this control strategy is impractical, as in many physical systems the power of the actuators are limited. When the number of agents in a network is very large, the information from the neighbors exceeds the saturation value of the actuators.

Motivated by the above mentioned considerations, in this paper we investigate finite-time control of higher order nonlinear agent dynamics subject to input saturations. In this paper, we first introduce a class of saturation functions based on it a distributed finite time bounded control protocol is obtained for higher order nonlinear MAS. By applying the homogeneous theory for stability analysis, a nominal control is obtained such that all the states of the followers can converge to that of the leader in finite time. By designing a suitable sliding surface a switching control is obtained to nullify the effect of the uncertainties in the agent dynamics.

The rest of this paper is organized as follows. Section 2 and 3 provides mathematical preliminaries and problem formulation. Section 4 presents homogeneous finite time consensus control with actuator saturation. An example is given in Section 5 to verify the theoretical analysis, which is followed by the conclusion in Section 6.

## 2. MATHEMATICAL PRELIMINARIES

In this section, we present some preliminary notations on finite-time stability, graph theory and homogeneous finite-time consensus of MAS to be used throughout the paper, and then formulate the finite-time consensus tracking problem of higher-order MAS with actuator saturation.

**Definition:Hong (2002):** Consider the system

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in \Re^n \tag{1}$$

where $f : U_0 \to \Re^n$ is continuous in an open neighborhood $U_0$ of the origin. Let $(r_1, ..., r_n) \in \Re^n$ with $r_i > 0, i = 1, ..., n$ and $f(x) = [f_1(x), ..., f_n(x)]^T$ be a continuous vector field. Vector function $f(x)$ is said to be homegeneous of degree $\kappa \in \Re$ with respect to $(r_1, ..., r_n)$ if for a given $\epsilon > 0$, $f_i(\epsilon^{r_1}x_1, ..., \epsilon^{r_n}x_n) = \epsilon^{\kappa + r_i} f_i(x), i = 1, ..., n, \forall x = [x_1, ..., x_n]^T \in \Re^n$. System (1) is said to be homogeneous if $f(x)$ is homogeneous.

**Lemma: Bhat and Bernstein (2005):** Suppose $\dot{x} = f(x)$ is homogeneous of degree $\kappa$. The origin of the system is finite-time stable if the origin is asymptotically stable and the system is with a negative homogeneity i.e. $\kappa < 0$.

### *2.1 Graph Theory*

Consider a MAS consisting of one leader and $N$ followers. The communication topology between agents, is modeled by a weighted directed graph $\mathcal{G} = \{\nu, \varepsilon, \mathcal{A}\}$ , where $\nu = \{0, 1, 2, .., N\}$ is the vertex set of the agents, node i represents the ith agent and $\varepsilon = \{(i, j) \subseteq \nu \times \nu\}$, represents the set of edges. The weight adjacent matrix is $\mathcal{A} = (a_{ij} \geq 0) \in \Re^{(N\times1)\times(N\times1)}$, where $(i, j) \in \varepsilon \Leftrightarrow a_{ij} = a_{ji} = 1$, otherwise $a_{ij} = a_{ji} = 0$ and $a_{ii} = 0$ for all $i \in \nu$ because $(i, i) \notin \varepsilon$. Therefore $\mathcal{A}$ is symmetric. For the leader-follower MAS, another graph $\bar{\mathcal{G}}$ can be considered to associate the system consisting of $N$ followers with the leader. The leader adjacency matrix is defined as $\bar{\mathcal{B}} = [b_1, b_2, \ldots, b_N]^T \in \Re^N$ with the adjacency element $b_i > 0$ if agent $i$ is a neighbor of the leader, otherwise $b_i = 0$. The followers can receive information from the leader, but cannot send information to the leader.

## 3. PROBLEM FORMULATION

Suppose the ith follower is governed by nth order dynamics as,

$$\begin{aligned} \dot{x}_i &= F_i(x_i, t) + G_i(x_i, t)u_i \quad i = 1, 2, ..., N \\ y_i &= h_i(x_i) \end{aligned} \tag{2}$$

where $x_i \in \Re^n$, $u_i \in \Re$, and $F_i(x_i, t) = F_0(x_i, t) + \Delta F_i(x_i, t)$ and $G_i(x_i, t) = G_0(x_i, t) + \Delta G_i(x_i, t)$ are $n$ dimensional vector fields. It is assumed that $F_0(x_i, t)$ and $G_0(x_i, t)$ are known functions, while $\Delta F_i(x_i, t)$ and $\Delta G_i(x_i, t)$ are unknown bounded uncertainties.

The Lie derivative of the output function $h_i(x_i)$ with respect to the vector field $F_i(x_i, t)$ can be obtained as follows:

$$L_{F_i} h_i(x_i) = \frac{\partial h}{\partial x} F_i(x_i, t) \tag{3}$$

Also the Lie derivative of $L_{F_i} h_i(x_i)$ with respect to the vector field $G_i(x_i, t)$ can be defined as:

$$L_{G_i} L_{F_i} h_i(x_i) = \frac{\partial}{\partial x}(L_{F_i} h_i(x_i)) G_i(x_i, t) \tag{4}$$

Sice the follower(2) has a relative degree, $r = n$, therefore one can easily obtain:

$$\begin{aligned} &L_{G_i} L_{F_i}^{k-1} h_i(x_i) = 0 \quad \forall k = 1, 2..., n-1 \\ &L_{G_i} L_{F_i}^{n-1} h_i(x_i) \neq 0 \end{aligned} \tag{5}$$

Using above, the $nth$ derivative of the output can be obtained as:

$$y_i^n = L_{F_i}^n h_i(x_i) + L_{G_i} L_{F_i}^{n-1} h_i(x_i) u_i \tag{6}$$

So the ith follower(2) can be transformed as:

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ &\vdots \\ \dot{x}_{in} &= f_i(x_i) + g_i(x_i)u_i \quad i = 1, 2, ..., N \\ &= f_{i0}(x_i, t) + g_{i0}(x_i, t)u_i + \underbrace{\Delta f_i(x_i, t) + \Delta g_i(x_i, t)u_i}_{\nabla_i} \\ y_i &= x_{i1} \end{aligned} \tag{7}$$

where $f_i = L_{F_i}^n h_i$ and $g_i = L_{G_i} L_{F_i}^{n-1} h_i \neq 0$ are the Lie derivatives. It is also assumed that, $f_i(x_i, t) = f_{i0}(x_i, t) + \Delta f_i(x_i, t)$ and $g_i(x_i, t) = g_{i0}(x_i, t) + \Delta g_i(x_i, t)$. $\Delta f_i(x_i, t)$ and $\Delta g_i(x_i, t)$ are unknown uncertainties and external