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IFAC-PapersOnLine 49-3 (2016) 291–296

Traffic Flow Model Validation Using METANET, ADOL-C and RPROP

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Abstract: Macroscopic traffic flow model calibration is an optimisation problem typically solved by a derivative-free population based stochastic search methods. This paper reports on the use of a gradient based algorithm using automatic differentiation. The ADOL-C library is coupled with the METANET source code and this system is embedded within an optimisation algorithm based on RPROP. The result is a very efficient system which is able to be calibrate METANET's second order model by determining the density and speed equation parameters as well as the fundamental diagrams used. Information obtained from the system's Jacobian provides extra insight into the system dynamics. A 22 km site is considered near Sheffield, UK and the results of a typical calibration and validation process are reported.

Keywords: Road traffic, calibration, parameter estimation, optimization, gradient methods.

1. INTRODUCTION

In Poole and Kotsialos (2012) an optimisation formulation was introduced for the macroscopic traffic flow model calibration problem which was solved by means of a genetic algorithm. The METANET Messmer and Papageorgiou (1990); Kotsialos et al. (1998, 2002) model was employed, treated as a simulation black box. An additional requirement was the automatic spatial assignment, i.e. determining the location and extension, of fundamental diagrams (FD). The motivation behind this is that current calibration practice either uses expert engineering opinion to make a decision about the FD or use a separate FD for every discrete road segment resulting from the model's discretisation rules. In the first case, intuition, past experience, visual inspection and preliminary data analysis result to an *ad-hoc* approach leading away from systems that embed knowledge in their own structure and the display of more intelligent forms of automation, Kotsialos and Poole (2013). In the latter case, overparametrisation is a clear risk since typically three parameters are necessary for defining a FD.

The problem formulation suggested in Poole and Kotsialos (2012) allows the arbitrary selection of FD location for homogeneous road stretches, which themselves are split into segments, but also penalised the variance between their parameters. The rationale behind this penalisation is that by treating the FD as an extensive quantity whose start and end are decision variables in an optimisation problem, the parameter variance penalty will result to solutions that favour similar FD. This kind of similarity was employed as guidance when validating the large scale model of the Amsterdam motorway networks, Kotsialos et al. (1998, 2002). An additional constraint imposed a maximum number of FD to be used for a site. It is left up to the optimisation algorithm to decide how many FD to be used and over which area to place them.

FD are aggregate descriptions of the infrastructure-vehicle-driver system. The variation in capacity and free speeds observed in real data are projections of the same traffic flow adapting to local inhomogeneities, e.g. drop of lanes, or different traffic composition. Variations of that system should be reflected on the FD but all FD model the same traffic flow process. This does not mean that the optimisation algorithm will equate all FD, since error minimisation is still the dominant objective.

The optimisation problem as formulated in Poole and Kotsialos (2012) is a nonlinear mixed integer optimisation problem. A genetic algorithm was used there in order to demonstrate the soundness of the approach. Based on this work, a more detailed calibration work using classic and recent variants of particle swarm optimisation (PSO) and cuckoo search algorithms was reported in Poole and Kotsialos (2016). These evolutionary algorithms were used for calibrating the Heathrow site used in Poole and Kotsialos (2012) in addition to a road stretch near Sheffield, which is considered here as well. The results reported demonstrate the validity of the approach. Optimal parameters were determined capturing the essential characteristics of the underlying traffic dynamics as was shown in the ensuing model validation, see Poole and Kotsialos (2016) for more details.

All the calibration methods used there are population based treating the METANET simulator as a simple executable invoked for each fitness function evaluation. This approach follows the common choice made regarding the optimisation algorithm used for model parameter estimation, see e.g. Spiliopoulou et al. (2014) and Ngoduy and Maher (2012). Here, a gradient based optimisation method is introduced for solving this calibration problem. Extra information that becomes available from the process of calculating partial derivatives is highlighted as well. The gradient calculation is performed by use of the automatic

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.049

differentiation algorithm ADOL-C, Walther and Griewank (2012).

Section 2 provides a brief overview of the METANET model. Section 3 outlines the optimisation problem formulation. A brief site description and overview of available data are given in section 4 and results are discussed in section 5. Section 6 concludes this paper providing the key areas of future work.

2. METANET MODEL OVERVIEW

METANET is a well known macroscopic traffic flow model. A road network is represented as a directed graph consisting of nodes and links. Links represent homogeneous road sections, where the number of lanes is a constant and there is no significant change of curvature or gradient. Nodes are connected by links and are used at places where the geometry of the motorway changes or at on-/off-ramp junctions. Traffic enters via origin links and leaves through destination links.

Time is discretised globally with a time step T and the time horizon is K steps. Each motorway link m is discretised into N_m segments of equal length L_m. The variables describing traffic conditions in segment i of link m, at time instant $t = kT$, $k = 0, 1, \dots, K$, are the traffic density $\rho_{m,i}(k)$ (veh/km/lane) of a link m with λ_m lanes, the mean speed $v_{m,i}(k)$ (km/h) and the traffic flow $q_{m,i}(k)$ (veh/h). The discrete time motorway second order traffic flow model is the following.

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m\lambda_m}[q_{m,i-1}(k) - q_{m,i}(k)] \quad (1)$$

$$q_{m,i}(k) = \rho_{m,i}(k)v_{m,i}(k)\lambda_m \quad (2)$$

$$\begin{aligned} v_{m,i}(k+1) = {} & v_{m,i}(k) + \frac{T}{\tau}\{V[\rho_{m,i}(k)] - v_{m,i}(k)\} \\ & + \frac{T}{L_m} v_{m,i}(k)[v_{m,i-1}(k) - v_{m,i}(k)] \\ & - \frac{\nu T}{\tau L_m}\frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa} \end{aligned} \quad (3)$$

where ν and κ are speed equation parameters and $V[\rho_{m,i}(k)]$ is the FD given by

$$V[\rho_{m,i}(k)] = v_{f,m} \cdot \exp\left[-\frac{1}{\alpha_m}\left(\frac{\rho_{m,i}(k)}{\rho_{cr,m}}\right)^{\alpha_m}\right] \quad (4)$$

where $\rho_{cr,m}$ is the critical density of link m and α_m a parameter.

In order to account for speed drops due to on-ramp inflow the term $-\delta T q_\mu(k)v_{m,1}(k)/\ (L_m\lambda_m(\rho_{m,1}(k) + \kappa))$ is added at (3), where δ is a constant parameter, μ the merging link and m is the leaving link. This term is included only when the speed equation is applied to the first segment of the downstream link m. Speed decreases due to weaving is accounted by adding the term $-\phi T\Delta\lambda\rho_{m,N_m}(k)v_{m,N_m}(k)^2/(L_m\lambda_m\rho_{cr,m})$ to (3), where $\Delta\lambda$ is the reduction in the number of lanes and ϕ is another parameter. Constraints are imposed in the form of a minimum speed $v_{\min}$ and a maximum density $\rho_{\max}$.

Traffic volume measurements at origins over the period of K steps are required. Speed measurements can also be used to better inform the model dynamics, but they are not necessary. In order for the speed equation to be applied at destinations s, measurements of the density trajectories $\rho_s(k)$ over the entire time horizon are provided as boundary conditions as well. For a full description of the METANET, see Messmer and Papageorgiou (1990) or its manual, METANET (2008).

3. OPTIMISATION PROBLEM FORMULATION

Equations (1)–(4) applied on an arbitrary motorway network can be expressed in the following discrete dynamic state-space system form

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{d}(k); \mathbf{z}]. \quad (5)$$

The state vector consists of the density and mean speed of every link segment, i.e.

$$\mathbf{x} = \left[\rho_{1,1}\, v_{1,1} \dots \rho_{M_1,N_{M_1}}\, v_{M_1,N_{M_1}}\right]^T \quad (6)$$

where M_1 is the number of motorway links in the network.

The disturbance vector $\mathbf{d}$ consists of the inflows q_o entering the system from entry points (origin links) like on-ramps or the upstream main site boundary and optionally the speeds v_o at these locations; the densities ρ_s at the exit locations (destination links) like off-ramps or downstream main site boundaries; and the turning rates β_n^μ at every split node n, where μ is the main out-link. Hence,

$$\mathbf{d} = \left[q_1\, v_1\, \dots q_{M_2}\, v_{M_2}\rho_1 \dots \rho_{M_3}\beta_1^{\mu_1} \dots \beta_{M_4}^{\mu_{M_4}}\right]^T \quad (7)$$

where M_2 is the number of origins, M_3 the number of destinations, M_4 the number of split junctions.

$\mathbf{z} \in \mathbb{R}^\Gamma$ consists of the model parameters as encountered in the dynamic density (1), speed (3) and fundamental diagram (4) equations. It includes the network-wide global parameters of the maximum density $\rho_{\max}$, minimum speed $v_{\min}$ and the mean speed equation (3) parameters τ, ν, ϕ, δ and κ. It also contains parameters related to the fundamental diagram, i.e. v_f, α, and ρ_{cr}.

A set of measurements $\mathbf{y}$ from a number of locations along the motorway, are used for comparing reality and model output. The resulting minimisation problem is

$$\min_{\mathbf{z}} J[\mathbf{x}(k), \mathbf{y}(k)] \quad (8)$$

subject to

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{d}(k); \mathbf{z}],\ \mathbf{x}(0) = \mathbf{x}_0 \quad (9)$$

$$\mathbf{z}_{\min} \leq \mathbf{z} \leq \mathbf{z}_{\max} \quad (10)$$

where $J[\mathbf{x}(k), \mathbf{y}(k)]$ is a suitable error function and $\mathbf{z}_{\min}$ and $\mathbf{z}_{\max}$ are the lower and upper bounds, respectively, of $\mathbf{z}$'s elements. The evaluation of J at $\mathbf{z}$ requires the forward integration of (9) given as input the measurements of $\mathbf{x}_0$ and $\mathbf{d}(k)$.

Let $\widehat{N}$ the number of FDs used; each one's parameters ρ_{cr}, α and v_f are included in $\mathbf{z}$, i.e.

$$\mathbf{z} = \left[\tau\, \kappa\, \nu\, \rho_{\max}\, v_{\min}\, \delta\, \phi\, v_f^1\, \alpha^1\, \rho_{cr}^1\, \dots v_f^{\widehat{N}}\, \alpha^{\widehat{N}}\, \rho_{cr}^{\widehat{N}}\right]^T. \quad (11)$$

When $\widehat{N} = 1$ a single fundamental diagram is used. If $\widehat{N} = M_1$ then every link has its own FD; this is the case for