Dynamic Model for estimating the Macroscopic Fundamental Diagram ⋆

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Abstract: The Macroscopic Fundamental Diagram (MFD) relates the number of circulating vehicles (or accumulation) to a neighbourhood's average speed or flow. In theory the MFD has a well-defined maximum which remains invariant over time. Recent studies, however, suggest that in practice this is not the case, and the MFD does present a variations over time. These variations in the MFD render traffic simulations, modelling and control schemes inaccurate, as these tools do not capture the dynamic nature of the MFD. This paper presents a dynamic model for estimating the MFD, so it does capture the MFD's time varying nature. A mathematical Kalman-filter based framework for solving the model and estimating the MFD are also presented. The application of the method on a small scale example shows the potential of the method.

Keywords: Macroscopic Fundamental Diagram (MFD); Traffic Network State; Dynamic Estimation; Traffic Control; Kalman-Filter

1. INTRODUCTION

Transportation infrastructure systems are critical to serving the needs of millions of people. Traffic congestion represents one of the major obstacles to an efficient operation. In 2011, congestion caused urban Americans to travel 5.5 billion hours more and to purchase an extra 2.9 billion gallons of fuel for a congestion cost of $121 billion, 0.7% of the US GDP Schrank et al. (2012). This problem will only worsen as the growth in demand has far outstripped the rate of installed highway capacity. In fact, the US Federal Highway Administration estimates that the number of vehicle miles traveled increased by 76% from 1980 to 1999, while total miles of installed highway capacity increased by only 1.5% Noland and Cowart (2000). These challenges are further exacerbated by population growth, concentrated demand due to urbanization, and depletion of natural resources Rosenberg et al. (2010). Thus, arises the need for better understanding and utilization of the transportation network.

Since the seminal work of Greenshields et al. (1933), the fundamental diagram has become one of the most common tools for modelling a road segment and it flow-density, speed-flow, and speed-density relations (e.g., Dervisoglu et al. (2009); Helbing (2009); Wu et al. (2011)). The fundamental diagram was the base for many studies, starting from the work of Lighthill and Whitham (1955) and Richards (1956), through the work of Gerlough and Huber (1975) all the way to the work on the cell-transmission model (CTM) Daganzo (1994, 1995), which adopted a simplified fundamental diagram. All the aforementioned work sought the fundamental diagram to time invariant.

Recent advances in computational, sensory and communication technologies have enabled the analysis of a network segment, rather than a single stretch. To this end, the following studies: Daganzo and Geroliminis (2008); Geroliminis and Daganzo (2007, 2008) showed, through the moving observer method, that the fundamental diagram can serve as a tight bound for the average flow-density states of any urban street or network. This work essentially has extended the use of the fundamental diagram from a single

road segment to a network. They dubbed this extension as the Macroscopic Fundamental Diagram (MFD). Using the MFD concept, traffic control is simplified enormously as it offers an aggregate model approach that considers the traffic dynamics of a large urban area. Their analysis, however, assumed time invariant road conditions and homogenous spatial distribution of vehicle density over the network Geroliminis and Sun (2011).

While the fundamental diagram is sought to be time invariant, many studies have shown that this is not the case for the MFD. Ji et al. (2010) showed, by utilising a microscopic simulation model, that the MFD shape is a property not only of the network itself but also of the applied traffic control measures, congestion and traffic demand. Thus, traffic itself has an effect on the concurrent MFD shape. Mazloumian et al. (2010) examined vehicle density and its variability and its effect on urban traffic network performance. They found the variability to be a key factor in the process. Saberi and Mahmassani (2012) investigated the impact of the spatial and temporal distribution of congestion in a network on the shape and properties of the flow-density relation. The research showed that the maximum network average flow is not a constant value but varies across different days. In addition, for the same value of average network occupancy, a variation of occupancy was observed. The variation was attributed to the formation of fragmented queues and traffic instability. Variability was also observed when the spatial distribution of densities over the network was inhomogeneous and the average network occupancy remained consistently high and roughly unchanged for successive time intervals. Knoop et al. (2012) recapitulated this notion and acknowledged that the MFD does change over time. In a later work Knoop et al. (2013) the impact of traffic dynamics on the MFD was explored. The latter showed that the network flow is not constant for a fixed number of vehicles as predicted by the MFD, but decreases due to local queuing and spill back processes around the congestion. Following the above body of work, we conclude that the MFD is time variant and is affected by time variant factors such as the concurrent traffic conditions.

One of the first applications of the MFD were its utilization in traffic network dynamic control. Zheng et al. (2012) presented a dynamic pricing scheme, where tolls are controlled by an MFD. Geroliminis et al. (2013) used an MFD-based control mechanism to govern traffic flow between two urban regions such that the number of trips that reach their destinations is maximized. Zhang et al. (2013) presented the use of the MFD for achieving optimal adaptive traffic signal system. While all these applications present an unprecedented tool for managing and controlling transportation facilities, they all assumed time invariant MFD. Thus, the dynamic nature of the MFD, as traffic state changes, has not been regarded. Using inaccurate (or outdated) MFD to describe the network in question, results in a suboptimal control and underutilization of the network. Therefore, there is a great need to generate a mechanism that follows traffic conditions and distribution over the network and updates the MFD accordingly.

In this paper, a new mechanism to update the MFD in an on-line fashion, through polynomial Kalman filter Zarchan and Musoff (2005), is presented. Such a filter allows to

avoid the large computational cost usually required by other approaches such as extended Kalman or particle filters. Further, the polynomial Kalman Filter allows for capturing the time varying nature of the MFD. Previous studies that aimed at estimating the MFD Daganzo and Geroliminis (2008); Geroliminis and Daganzo (2008, 2007); Xie et al. (2013); Gayah and Dixit (2013); Knoop and Hoogendoorn (2013) did not account for temporal variability, which manifested itself in a scattered pattern. For example, Gayah and Dixit (2013) introduced a methodology to estimate the MFD from in traffic agents monitoring. Their work was shown accurate when the network is congested, and less so, when the network was uncongested. Ojeda et al. (2013) suggested Kalman filtering for estimating road density. Their work estimated the density state given MFD fixed model and stochastic perturbations. Therefore we conclude that the MFD does change with traffic and when the network is uncongested, there is a small number of agents to get accurate estimation of traffic.

Thus, the contribution of this paper is twofold: (i) it presents a model for a continuous updating of the MFD in traffic control applications, as traffic changes; and (ii) it introduces a simple, yet efficient, mechanism for updating the MFD as measurements acquired from the network are gathered. Finally, a demonstration of the suggested scheme is given on a real-life network. The paper is organized as follows. The MFD concept is presented in Section 2. In Section 3, the design of the MFD based on the polynomial Kalman filter is addressed, whereby a various estimation schemes are proposed and discussed. Then Section 4 describes the data processing. Several key issues concerning MFD estimation are carefully investigated in Section 5. Finally, Section 6 draws some conclusions and future work.

2. MACROSCOPIC FUNDAMENTAL DIAGRAM

The macroscopic fundamental diagrams have been proven to exist in small networks, relating the local flow and density Geroliminis and Daganzo (2007). A typical schematic MFD is illustrated in Figure 1. The MFD describes the
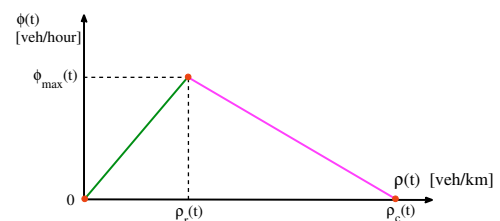

Fig. 1. Macroscopic fundamental diagram.

transition of the road network between three states: *Free flow* - When the density is low and the the flow $\phi(t)$ is undisturbed. This stage is described by the green part shown in Figure 1. With the increase of the number of vehicles, $\phi(t)$ rises up to the maximum, indicated by the point $(\rho_r(t), \phi_{max}(t))$, called the *sweet spot* or *critical density*. As the number of vehicles further increases, passengers will