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Convexity and Robustness of Dynamic Network Traffic Assignment for Control of Freeway Networks

Giacomo Como * **Enrico Lovisari** ** **Ketan Savla** ***

* *Department of Automatic Control, Lund University, BOX 118, 22100, Lund, Sweden.* giacomo.como@control.lth.se.
** *Univ. J. Fourier and GIPSA-lab, CNRS, Grenoble, France.* enrico.lovisari@gipsa-lab.fr.
*** *Sonny Astani Department of Civil and Environmental Engineering, University of Southern California, Los Angeles, CA.* ksavla@usc.edu

Abstract We study System Optimum Dynamic Traffic Assignment (SO-DTA) for realistic traffic dynamics controlled by variable speed limits, ramp metering, and routing controls. We consider continuous-time cell-based Dynamic Network Loading models that include the Cell Transmission Model with FIFO rule at the diverge junctions as well as non-FIFO diverge rules. We consider SO-DTA formulations in which the total inflow into and the total outflow from the cells are independently constrained by concave supply and demand functions, respectively, thus preserving convexity. We design open-loop controllers that guarantee that the optimal solutions are feasible with respect to realistic traffic dynamics, and we develop this methodology for two variants of the SO-DTA problem, one of which accounts for exogenous turning ratios. Robustness of the system trajectory under the proposed controllers is evaluated by deriving bounds on the deviations induced by perturbations of the initial condition and of the external inflows.

Keywords: Traffic flow control, cell-transmission model, dynamic traffic assignment, robustness.

1. INTODUCTION

Dynamic Traffic Assignment (DTA), introduced in Merchant and Nemhauser (1978a), has attracted a large amount of attention by the transportation research community as a standard framework for control of freeway networks, see Peeta and Ziliaskopoulos (2001) for an overview. The focus of this paper is on System Optimum Dynamic Traffic Assignment (SO-DTA), that aims at minimizing a system-level cost function over a planning horizon employing variable speed limits, ramp metering, and routing control, and subject to realistic traffic dynamics. The latter are modelled via a combination of features of the Cell Transmission Model (CTM), Daganzo (1994), and a general Dynamic Network Loading model, Cascetta (2009), that include both FIFO and non-FIFO policies.

While variable speed limits, ramp metering and routing are the most common forms of control for freeway networks, they are not often incorporated in DTA formulations as they typically lead to non-convex and computationally expensive optimizations, unsuitable for real-time applications. Nonetheless, there have been efforts to relax non-convex features of traffic dynamics and control variables in DTA. For linear demand and affine supply functions in CTM, and linear cost, Ziliaskopoulos (2000) shows that the SO-DTA problem for single destination networks with no control variables can be cast as a linear program under the relaxation where the total inflow into and the total outflow from the cells are independently constrained to be upper bounded by supply and demand, respectively. While the optimal solution to the relaxed problem obviously has no higher cost than the original formulation, it is not clear if it can be implemented in the actual traffic dynamics.

The existing efforts to investigate the implementability of the optimal solution are few and limited to specific types of demand and supply functions, network structure, and DNL model. In Muralidharan and Horowitz (2012), the authors show that the optimal solution obtained under the relaxation proposed in Ziliaskopoulos (2000) can be realized exactly for traffic dynamics modeled by the link-node cell transmission model using ramp metering and variable speed limits, when demand functions are linear, supply functions are affine, and the network consists of a mainline with on- and off-ramps. The first goal of this paper is to substantially extend the state-of-the-art following our recent contribution Como et al. (2015a). We show that the SO-DTA with or without exogenous turning ratios can be written as a convex problem and that its optimal solution can be realized using a combination of ramp metering, speed limit and (when available) turning ratios prescription, for possibly nonlinear demand and supply functions under mild assumptions and for generic network topologies, under both FIFO and non-FIFO DNL models. This methodology is a generalization of the specific scenarios considered in Muralidharan and Horowitz (2012).

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The second goal objective of this paper is to study robustness of optimal solutions of the considered SO-DTA formulations with respect to uncertainties in initial condition and external inflow over the planning horizon. We provide bounds on the maximum perturbation of the system trajectory under the proposed open-loop controllers in terms of perturbations in initial condition and external inflows. We leverage a certain monotonicity property that is satisfied whenever the traffic dynamics is in free-flow, which we prove to be the case for the optimal trajectory and for trajectories close to it. The resulting bounds are applicable for values of perturbations that are relatively larger than those obtained through sensitivity analysis of ordinary differential equations. The proposed bounds are shown to be in close agreement with simulated trajectories on the benchmark network in Ziliaskopoulos (2000). While we limit our analysis to perturbations on initial conditions and external inflows, future research include perturbations in demand, supply and turning ratios, therefore analysing the robustness of the proposed control when drivers do not follow the nominal traffic model.

The paper is organized as follows. Section 2 summarizes the convex formulations of SO-DTA introduced in Como et al. (2015a) and discusses their feasibility with respect to traffic dynamics for general DNL models, including FIFO and non-FIFO policies. Section 3 provides perturbations bounds on DTA solution due to uncertainties in initial condition and external inflows. Section 4 presents numerical simulations. The proofs of technical results can be found in the extended version of this work Como et al. (2015a).

2. CONVEX FORMULATIONS AND FEASIBILITY OF CONTINUOUS-TIME DTA

This section summarizes the framework introduced in Como et al. (2015a) for convex continuous-time SO-DTA and their feasibility for general DNL models including FIFO and non-FIFO policies.

2.1 Convex Formulations of continuous-time SO-DTA

We describe the topology of the transportation network as a directed multi-graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where nodes represent junctions and links represent cells. The head and tail nodes of a cell i are denoted by τ_i and σ_i, so i is directed from σ_i to τ_i. One particular node $w \in \mathcal{V}$ represents the external world, with cells i such that $\sigma_i = w$ and cells i such that $\tau_i = w$ representing on-ramps and off-ramps, respectively. The sets of on-ramps and off-ramps will be denoted by $\mathcal{R}$ and $\mathcal{R}^o$, respectively. The topology is typically illustrated by omitting such external node w and letting on-ramps have no tail node and off-ramps have no head node. (See Figure 1.) The set $\mathcal{A} = \{(i,j) \in \mathcal{E} \times \mathcal{E} : \tau_i = \sigma_j \neq w\}$ is the set of all pairs of adjacent (consecutive) cells.

The dynamic state of the network is described by a time-varying vector $x(t) \in \mathcal{R}^{\mathcal{E}}$ whose entries $x_i(t)$ represent the mass (or traffic volume) in cell $i \in \mathcal{E}$ at time t. The inputs of the network are the inflows $\lambda_i(t) \geq 0$ at on-ramps $i \in \mathcal{R}$. Conventionally, we set $\lambda_i(t) \equiv 0$ for any $i \notin \mathcal{E} \setminus \mathcal{R}$, and stack up all the inflows in a vector $\lambda(t) \in \mathbb{R}^{\mathcal{E}}$. The physical constraints are captured by demand functions $d_i(x_i)$ and supply functions $s_i(x_i)$, returning the maximum possible

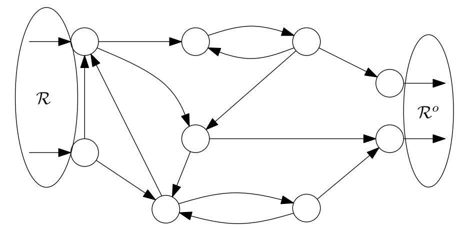

Figure 1. A multi-origin multi-destination cyclic network.

outflow from cells $i \in \mathcal{E}$ and the maximum possible inflow in non on-ramp cells $i \in \mathcal{E} \setminus \mathcal{R}$, respectively, as a function of the current mass x_i. Conventionally, we put $s_i(x_i) \equiv +\infty$ at all on-ramps $i \in \mathcal{R}$. The demand functions are assumed to be continuous, non-decreasing, and such that $d_i(0) = 0$, while the supply functions are assumed to be continuous, non-increasing, and such that $s_i(0) > 0$, with $x_i^{\text{jam}} = \inf\{x_i > 0 : s_i(x_i) = 0\}$ denoting cell i's jam mass. Crucially, we focus on the case where all demand and supply functions are concave in their argument.

We assume that an initial value $x_i^0 \geq 0$ on every cell $i \in \mathcal{E}$ is given, and we aim to minimize the integral of a running cost $\psi(x)$, function of the entire vector of mass x, over a time horizon $[0, T]$. We assume that $\psi(x)$ is convex in x, nondecreasing in each entry x_i, and such that $\psi(0) = 0$. A particularly relevant case is a separable cost, e.g., when $\psi(x) = \sum_{i \in \mathcal{E}} \psi_i(x_i)$, for convex non-decreasing costs $\psi_i(x_i)$ on each $i \in \mathcal{E}$, with $\psi_i(0) = 0$. We will use the following optimization variables, all function of time: x_i, y_i, and z_i stand, respectively, for the mass on, the inflow in, and the outflow from, cell $i \in \mathcal{E}$; f_{ij} stands for the flow between two cells i, j; and μ_i is the the out-flow from an off-ramp $i \in \mathcal{R}^o$ that leaves the network.

The basic version of the SO-DTA problem can be formulated as the following optimization problem

$$\min \int_0^T \psi(x(t))\mathrm{d}t \tag{1}$$

such that,

$$x_i(0) = x_i^0 \,, \qquad i \in \mathcal{E} \,, \tag{2}$$

and for all $t \in [0, T]$,

$$\dot{x}_i(t) = y_i(t) - z_i(t) \,, \qquad i \in \mathcal{E} \,, \tag{3}$$

$$\left.\begin{aligned} y_i(t) &= \lambda_i(t) + \sum_{j \in \mathcal{E}} f_{ji}(t) \,, \\ z_i(t) &= \mu_i(t) + \sum_{j \in \mathcal{E}} f_{ij}(t) \,, \end{aligned}\right. \qquad i \in \mathcal{E} \,, \tag{4}$$

$$\mu_i(t) \geq 0 \,, \quad f_{ij}(t) \geq 0 \,, \qquad i, j \in \mathcal{E} \,, \tag{5}$$

$$\begin{aligned} f_{ij}(t) &= 0 \qquad (i,j) \in \mathcal{E} \times \mathcal{E} \setminus \mathcal{A} \,, \\ \mu_i(t) &= 0 \qquad i \in \mathcal{E} \setminus \mathcal{R}^o \,, \end{aligned} \tag{6}$$

$$y_i(t) \leq s_i(x_i(t)) \,, \qquad z_i(t) \leq d_i(x_i(t)) \,, \quad i \in \mathcal{E} \,. \tag{7}$$

Let us now consider an exogenous, possibly time-varying, routing matrix R, which is a nonnegative $\mathcal{E} \times \mathcal{E}$ matrix satisfying the network topology constraints

$$R_{ij} = 0 \,, \qquad (i,j) \in (\mathcal{E} \times \mathcal{E}) \setminus \mathcal{A} \,, \tag{8}$$

$$\sum_{j \in \mathcal{E}} R_{ij} = 1 \,, \qquad i \in \mathcal{E} \setminus \mathcal{R}^o \,. \tag{9}$$

The matrix R is to be interpreted as describing the drivers' route choices, with its entries R_{ij}, sometimes referred to