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A Practical Framework for the Analysis of Average Consensus Problem: Laplacian Generator and Analyzer Algorithm

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Abstract: This paper provides a topology based analysis for the average consensus problem regarding various configurations and possibilities. Consensus problem which can be defined as the agreement of the agents in a multi-agent topology is a common problem in various fields of science, technology and engineering. The main subject focused on within the scope of this study, is the selection of the topology regarding various performance criteria for the agents to reach consensus. Basic concepts of multi-agent systems are introduced along with an algebraic background. Graph Theory and its properties are also covered as a basis for understanding the behavior of multi-agent systems. A numerical approach in order to determine the structure of the connection topology that satisfies the predefined performance criteria, is proposed. In the end, case studies are included to verify the effectiveness of the proposed method.

Keywords: Consensus, Network Topology, Multi-Agent Systems, Average Consensus Problem, Laplacian Matrix, Numerical Topology Optimization

1. INTRODUCTION

Consensus problem's origin may set back to ancient times of humanity as it may be interpreted as social, political or a strategic problem. The very nature of the behavior of animals and human beings have been in scope for many philosophers as well as scientists. The consensus seen in the nature is often stunning and miraculous at some points. It is also referred to as swarm intelligence, collective behavior or herd behavior (Bouffanais, 2016). This alone was a great inspiration for both engineers and scientist to understand, model and realize the beauties of this deep concept. One significant example about the modelling of consensus in nature is the biologically motivated work of Vicsek et al., (1995) which models the flocking movement and alignment problem.

From the engineering point of view, consensus problem which can be informally defined as the problem of agreement of units in a multi-unit system. This problem consists of a series of challenges for multi-unit systems. It must be pointed out that the term multi-unit may refer to multi-agent systems and also to distributed systems which in fact is a problem of cooperation among certain units and decision making mechanisms (DeGroot, 1974).

The problems that may occur during this cooperation depend on various causes. For the sake of coordination, connection and cooperation between agents, the solution of the consensus problem is of great importance. Even though the problem of maintaining a reliable connection is important as it might seem, our focus in this paper is to satisfy the consensus, even in cases that some of these connections are lost.

It is proposed within the scope of this study that the boundaries for the numbers of connections to be maintained for the agents to reach the consensus can be determined using numerical approaches. For this purpose, firstly, a set of performance criteria in terms of consensus location and the speed of convergence to that point in the average consensus problem should be defined for different topologies. Here, the term topology refers to the relationship among agents in terms of information exchange.

Average consensus problem, as we will be referring to as the consensus problem throughout this paper is one of the most fundamental problems of consensus. In the average consensus problem the units which have the capability of evaluating information and making decisions which we refer to as agents must agree on a single point. This point will be referred to as consensus point which is indeed a saddle point for all agents and a point which these agents should go in order to reach consensus (DeGroot, 1974). It may be considered as a general state that each agent has and may converge upon the agreed value. One of the simplest example for this case, is a group of sensors in a room, each measuring an average of some local values (Avrachenkov et al., 2011). For that kind of situation, the average consensus problem is more about measured values such as humidity or temperature rather than location or speed.

In literature, distributed networks and reaching a consensus among decision making units problem was firstly started by the work of Borkar and Varaiya (1982) and Tsitsiklis (1984). The theoretical framework for consensus problems was introduced by the studies of R.Olfati-Saber and Murray

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(2003) and (2004) based on the works of Fax and Murray (Fax, 2001), (Murray and Fax, 2004).

One of the most useful areas of the consensus problem is the management in transportation systems. Platoon control is necessary for the intelligent and efficient coordination of vehicles. A group of heavy-duty vehicles can be put into formations, namely platoons, in order to increase the safety and fuel efficiency (Alam et al., 2015). This is achieved by forming a line of vehicles, keeping the intervehicular distances and reducing the aerodynamic drag force for the non-leading vehicles, just like the principle behind the flying bird formations. Driverless cars forming a platoon with a sensor based network topology is also a consensus problem which consists problems regarding collision avoidance, formation adjustment according to changing weather and road conditions, sustaining the formation under various disturbances (Wang et al., 2014).

2. CONSENSUS PROBLEM AND ITS FORMULATION

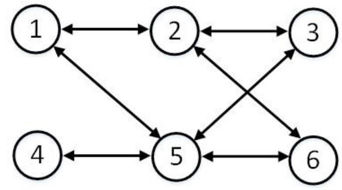
Graph Theory is an important tool for the analysis of consensus problems. It can be directly described by the Laplacian Matrix which involves all the necessary properties that we need to model a consensus problem. Some important concepts related with the formulation of such problems are:

- Agent: An individual robot or any unit that has a state along with the desire to reach consensus with other agents.
- Topology: The topology of agents is related to their connections among each other. Most common way to express a topology is by using the graph theory. Each node in the graph indicates an agent.
- Consensus Time: The time that consensus is reached by all agents.
- Consensus Position: The position of consensus which is related to the initial states of all agents and their information exchange relationships, namely their network topology.

Algebraic graph theory has a set of properties and definitions that are directly applicable to consensus problems. It is essential to know these properties in order to determine the outcome of the consensus problem and how its outcome is influenced by the type of the graph. In areas involving networks such as the consensus problem, networks are usually defined by symmetrical matrices and the solutions for the problems lie within the fundamentals of the Graph Theory (Steven, 2010).

An introductory example of forming a Laplacian Matrix is given in Figure 1.

Graph Representation



Degree Matrix

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

Adjacency Matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Laplacian Matrix

$$\begin{pmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & 0 & -1 \\ 0 & -1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & -1 & -1 & 4 & -1 \\ 0 & -1 & 0 & 0 & -1 & 2 \end{pmatrix}$$

Fig. 1. Graphical Representation and Determination of the Laplacian Matrix

A graph consists of vertices and edges in graph theory. In a consensus problem, the relationship between the agents are included in the Laplacian matrix. A Laplacian matrix is a square matrix consisting of an adjacency matrix and a degree matrix and it is obtained by subtracting the adjacency matrix from the degree matrix. Further properties of the Laplacian matrix such as being positive semi-definite and diagonally dominant are also the important properties that make the analysis of a network possible (Cvetkovic et al., 1998).

3. MODELLING THE CONSENSUS PROBLEM

It can be proposed that a graph representation alone is not enough to represent a complete average consensus problem. It is important to state that in order for agents to change their state values and solve an average consensus problem, a dynamical system which consists of integrators is essential (Olfati-Saber and Murray, 2004). A system representation formed by block diagrams was introduced by Olfati-Saber and Murray (Olfati Saber et al., 2007), as given in Figure 2.

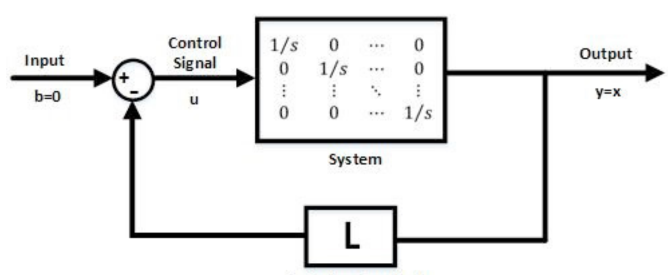


Fig. 2. Block diagram representation of an average consensus problem

The nature of the consensus problem can be modelled as an autonomous system with a state-space equation without an input. For the average consensus problem, convergence occurs around the average of the initial states of all agents if the communication graph is balanced (Olfati-Saber and Murray, 2004). Therefore the dynamics of the system can be written as (Olfati-Saber and Murray, 2004), (Lynch, 1997):