Coordination of Distributed Model Predictive Controllers for Constrained Dynamic Processes ⋆

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Abstract: In this paper, a coordinated-distributed model predictive control (MPC) scheme is presented for large-scale discrete-time linear process systems. Coordinated-distributed MPC control aims at enhancing the performance of fully decentralized MPC controllers by achieving the plant-wide optimal operations. The 'price-driven' decomposition-coordination method is used to adjust the operations of the individual processing units in order to satisfy an overall plant performance objective. Newton's method, together with a sensitivity analysis technique, are used to efficiently update the price in the price-driven decomposition-coordination method. The efficiency of the proposed control scheme is evaluated using a model of a fluid catalytic cracking process.

Keywords: Decomposition-coordination methods; Large-scale optimization; Optimal control theory.

1. INTRODUCTION

Since the late seventies, the design of chemical processes has evolved towards integrated operations that have increased plant's efficiency. The improvement in the design of chemical processes included, among other things, energy and mass integration, and the use of recycle streams. As a result, processes became more complex and processing units became more tightly interconnected. Control of such integrated large-scale processes has been typically performed with *decentralized* schemes because of the difficulties in implementation and maintenance of *centralized* control frameworks.

Centralized and decentralized control are two distinct control strategies. In centralized control, no real distinction is made among processing units. The centralized control framework is formulated as a monolithic control problem that incorporates all process variables with no unit-level decomposition. While a centralized strategy can lead to optimal plant-wide performance, it presents some disadvantages (e.g., the large-dimensionality of the control problem and lack of flexibility in terms of operation and maintenance), which make centralized control unsuitable for industrial processes. In decentralized control, each engineering unit is optimized separately by neglecting the interactions with the other units. The decentralized approach is the most commonly used in the industry because of its robustness and its resiliency to systems failures. Nevertheless, decentralized control does not generally lead to the desired plant-wide optimal operations (Lu (2003); Sun and El-Farra (2008)).

A compromise between centralized and decentralized control is desired in order to improve plant operations. *Distributed* control has emerged as a promising control strategy that can lead to the plant-wide optimal operations, while keeping manageable controllers for each subunit in the plant. In the distributed control framework, it is assumed that each subsystem computes its own optimal solution while considering all or certain degree of interactions with the other subsystems. To attain the desired control performance, information related to each subsystems' optimal solutions is generally exchanged among the subsystems. In this work, we present a coordinated-distributed model predictive control (CDMPC) framework for constrained dynamic processes. In CDMPC control, data is exchanged with each individual MPC controller via a 'coordinator' or 'master'.

1.1 Distributed MPC Control

Distributed MPC control has attracted the attention of many researchers in recent years. Dunbar and Murray (2004) formulated MPC platforms for nonlinear interacting subsystems (multi-vehicle formations) whose state variables are coupled in a single objective function. For linear interconnected systems, Venkat et al. (2005) proposed a communication-based MPC that can converge to a Nash equilibrium. The communication-based MPC was further improved by a cooperation-based MPC that leads to the Pareto optimal feasible solution. Cheng et al. (2008, 2007) proposed a coordinated scheme for MPC steady state target calculation based on Dantzig-Wolfe decomposition and price-driven coordination methods, respectively.

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The main contribution of this work is to propose the *price-driven* decomposition-coordination algorithm, as described in Cheng et al. (2007), for the control of constrained process systems whose dynamics are represented by discrete-time models. The CDMPC control scheme presented in this paper achieves the centralized optimal operations and can be implemented when step-response models are available for the process. Since our control formulation uses models obtained from step-test data, it does not need estimation of unavailable process variables (as it might be required when formulating MPC controllers based on state-space models). Furthermore, the proposed CDMPC control scheme allows for bias correction in the predicted outputs through feedback.

An illustration of CDMPC is shown in Fig. 1. The price-driven decomposition-coordination method is used in the formulation of the CDMPC controllers. In the price-driven decomposition-coordination method, the coordinator sets up a price, 'p', for the subsystems' interacting variables (Fig. 1). The price provided by the coordinator is then

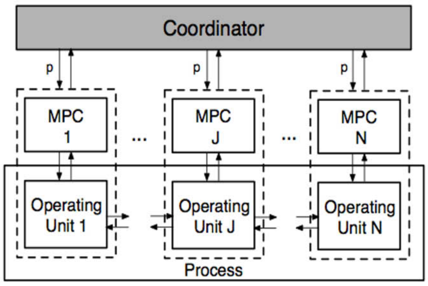

Fig. 1. Illustration of CDMPC Control

adjusted to alter the subunits' calculated control actions towards the overall plant optimum. In this work, the price, p, is updated based on Newton's method. An iterative procedure is established between the coordinator and the subunits until the desired plant-wide optimal solution is achieved.

2. CDMPC CONTROL FOR DYNAMIC PROCESS SYSTEMS

In this section, the CDMPC control scheme is presented. Since we consider the centralized performance as the ideal benchmark, we begin the CDMPC control formulation by decomposing the centralized control problem into N smaller subproblems that are easier to solve. Then, an efficient mechanism is used to achieve the same solution as the one obtained in the centralized control problem.

2.1 Process Model

Consider the overall plant process, modelled by step-response coefficients:

$$y_z(k+l) = \sum_{h=1}^{T-1}\sum_{w=1}^{r} S_{zw,h}\Delta u_w(k+l-h) + \sum_{w=1}^{r} S_{zw,T}\; u_w(k+l-T), \quad \forall z = 1,\dots,m, \tag{1}$$

where y_z $(\forall z = 1,...,m) \in \Re^m$ denote the process outputs; $u_w \in \Re^r$ and $\Delta u_w \in \Re^r$ $(\forall w = 1,...,r)$ denote the manipulated variables and the change in the manipulated variables, respectively. The coefficients $S_{11,h},...,S_{mr,h}$ represent the step-response coefficients for h^{th} time step. The step-response weight $S_{11,h}$ is the coefficient between Δu_1 and output y_1 for the h^{th} time step. In a similar manner, $S_{mr,h}$ is the coefficient between Δu_r and output y_m for the h^{th} time step.

2.2 Centralized MPC Formulation

For the centralized MPC implementation, it is convenient to arrange process model (1) in a matrix form as following:

$$\hat{Y}(k+1) = \mathcal{S}\Delta\hat{U}(k) + Y^0(k+1) + \hat{D}(k+1), \tag{2}$$

where the output variables, input variables and change in input variables predicted along the prediction horizon H_p and control horizon H_u are defined as:

$$\begin{cases} \hat{Y}(k+1) = [\hat{y}(k+1|k)^\top,...,\hat{y}(k+H_p|k)^\top]^\top, \\ \hat{y}(.) = [\hat{y}_1(.),...,\hat{y}_m(.)]^\top, \\ \Delta\hat{U}(k) = [\Delta\hat{u}(k|k)^\top,...,\Delta\hat{u}(k+H_u-1|k)^\top]^\top, \\ \Delta\hat{u}(.) = [\Delta\hat{u}_1(.),...,\Delta\hat{u}_r(.)]^\top, \hat{u}(.) = [\hat{u}_1(.),...,\hat{u}_r(.)]^\top. \end{cases} \tag{3}$$

The $m \times H_p$ vector of unforced responses $Y^0(k+1)$ is:

$$\begin{cases} Y^0(k+1) = [y^0(k+1)^\top,...,y^0(k+H_p)^\top]^\top, \\ y^0(.) = [y^0_1(.),...,y^0_m(.)]^\top. \end{cases} \tag{4}$$

The vector $\hat{D}(k+1)$ has been incorporated in (2) to correct through feedback the discrepancies between the measured and predicted outputs. The vector $\hat{D}(k+1)$ is defined as:

$$\hat{D}(k+1) = \underbrace{[I_m,...,I_m]^\top}_{H_p\ times}[y(k) - \hat{y}(k|k-1)],$$

where I_m is the $m \times m$ identity matrix. It is assumed that the difference between the measured and predicted outputs at time k remains constant throughout the prediction horizon.

In (2), the matrix of step-response coefficients $\mathcal{S}$ is defined as:

$$\mathcal{S} = \begin{bmatrix} S_1 & 0 & \dots & 0 \\ S_2 & S_1 & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ S_{H_u} & S_{H_u-1} & \dots & S_1 \\ \vdots & \vdots & \ddots & \vdots \\ S_{H_p} & S_{H_p-1} & \dots & S_{H_p-H_u+1} \end{bmatrix}, \tag{5}$$

where S_h is the $m \times r$ matrix of step-response coefficients for the h^{th} time step $(\forall h = 1,...,H_p)$:

$$S_h = \begin{bmatrix} S_{11,h} & S_{12,h} & \dots & S_{1r,h} \\ \vdots & \dots & \dots & \vdots \\ S_{m1,h} & \dots & \dots & S_{mr,h} \end{bmatrix}. \tag{6}$$

The centralized MPC controller is formulated to minimized the following objective function: