Boundary geometric control of co-current heat exchanger

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Abstract: A control strategy is proposed to control the internal fluid temperature at the outlet of a co-current heat exchanger by manipulating the inlet external fluid temperature. The dynamic model of the heat exchanger is given by two partial differential equations. Based on nonlinear geometric control, a state-feedback law that ensures a desired performance of a measured output defined as spatial average temperature of the internal fluid is derived. Then, in order to control the outlet internal fluid temperature, a control strategy is proposed where an external controller is introduced to provide the set point of the considered measured output by taking as input the error between the outlet internal fluid temperature and its desired set point. The validity of the proposed control design and strategy is examined in simulation by considering the tracking and perturbation rejection problems.

Keywords: distributed parameter system, partial differential equation, geometric control, characteristic index,PI controller, co-current heat exchanger.

1. INTRODUCTION

As a thermal device, heat exchangers are widely used in process industries both for cooling and heating operations. The dynamic behavior of the heat exchanger is modeled by a set of partial differential equations (PDE) that describe the spatio-temporal variation of the temperatures. Thus, the need to find the best operating conditions for the heat exchangers and to improve their effectiveness lead to take into account their distributed nature. In this context, good performances can be attained using more efficient control strategy based on the direct use of the distributed parameter model rather than a reduced or a lumped model (Ray, 1989; Christofides, 2001).

Heat exchangers can be classified into two major types according to their flow arrangement: co-current and counter-current heat exchangers. For the first one, the two fluids travel in the same direction. By contrast, for the second one, the fluids move in opposite directions.

In the control problem of tube heat exchangers, the variable which is manipulated, theoretically, is the thermal power at the inlet of the outer tube, i.e. grossly the product of a flow rate and a difference of temperature. In practice, to control the outlet temperature of a heat exchanger, two possible strategies which are not equivalent exist. The first one is to use the inlet temperature of the external fluid, while the second is to manipulate its flow rate.

When the flow rate is considered as a manipulated variable, if it becomes too low, the flow regime in the outer tube can be laminar instead of turbulent, which affects the parameters of the models, in particular the heat transfer coefficient (Xuan and Roetzel, 1993). So the tuning of the controller should vary with the flow rate, which is a difficult task (Abdelghani-Idrissi et al., 2001; Arbaoui et al., 2007). In addition, by manipulating the flow rate, a minimum bound is to set on this input. With the temperature as a manipulated input, it is possible to work at a constant large flow rate and the hydrodynamic regime is invariable. Physically, manipulating the temperature is almost possible if this latter is the outlet of a process with fast dynamics like plate heat exchangers. Potential flow rate variations will be assumed as a disturbance that affects the system and needs to be rejected by the designed controller.

Control of counter-current heat exchanger has attracted much attention, and several strategies are proposed based either on the PDE model or ODE model (see e.g. Maidi et al. (2008a) for more references) compared to the co-current heat exchanger for which few methods have been proposed in the literature. Derese and Noldus (1980) addressed the problem of controlling of the co-current heat exchanger using dynamical lumped parameter controllers designed based on technical frequency domain specifications. Based on the conjugate gradient method (CGM) of minimization, Huang and Yeh (2003) proposed an algorithm for determining an optimal external distributed heat-flux of a steady state co-current heat exchanger.

In this paper, a control strategy is proposed to control the outlet internal fluid temperature of a co-current heat exchanger by manipulating the inlet external fluid temperature. The designed approach is based on the use of the PDE model that describes the dynamic behavior. The idea is to design a state-feedback control that allows controlling the average temperature of the internal tube of the heat exchanger, assumed as the measured output. As it will be demonstrated, the direct design of a control law by considering the outlet temperature as the controlled variable is a difficult task due to the fact that the process is infinite-dimensional. Then, in order to control the outlet fluid temperature, a control strategy is proposed where a PI controller is introduced to provide the set point of the measured

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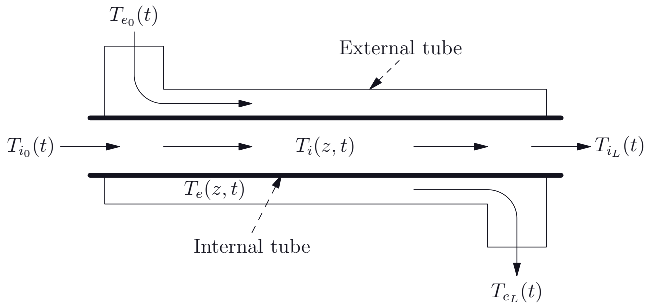

Fig. 1. co-current heat exchanger.

output (spatial average temperature). The design of the state-feedback control law makes use of the concept of characteristic index (Christofides and Daoutidis, 1996), which characterize the spatiotemporal interactions between the controlled and manipulated variables.

The paper is organized as follows. In section 2, the studied co-current heat exchanger is presented and its dynamic model given as a system of two PDEs. Section 3 concerns the formulation of the control problem and the design methodology. Section 4 is dedicated to simulation results concerning tracking and perturbation rejection problems. Finally, a conclusion ends the article.

2. CO-CURRENT HEAT EXCHANGER DYNAMIC MODEL

2.1 Description of the heat exchanger

The process studied in this work corresponds to a tubular co-current heat exchanger (Fig. 1). A fluid of constant density ρ_i and of heat capacity C_{p_i} flows through the internal tube of a heat exchanger, of length L, with a constant velocity v_i. This fluid enters at temperature T_{i_0} and exchanges heat with the an external fluid or non condensating vapor fluid, of constant density ρ_e and of heat capacity C_{p_e}, which flows in the same direction in the jacket with a velocity v_e. This fluid enters at temperature T_{e_0}. At the outlet of the exchanger, the internal fluid leaves at temperature T_{i_L}. In the present study, the internal and external cross sections S_i and S_e of the heat exchanger are supposed to be uniform and the surface area used for the heat transfer per unit length is $\mathcal{A}$. Both temperatures T_i of the internal fluid and T_e of the external fluid depend on time and spatial position along the tube.

The energy balance of the heat exchanger, after classical simplifying hypotheses (Ray and Ogunnaike, 1994), gives the following partial differential equation for the internal tube (internal fluid)

$$\frac{\partial T_i(z,t)}{\partial t} = -v_i \frac{\partial T_i(z,t)}{\partial z} + h_i \left[T_e(z,t) - T_i(z,t)\right] \qquad (1)$$

and the following partial differential equation for the jacket (external fluid)

$$\frac{\partial T_e(z,t)}{\partial t} = -v_e \frac{\partial T_e(z,t)}{\partial z} + h_e \left[T_i(z,t) - T_e(z,t)\right] \qquad (2)$$

where $h_i = \dfrac{U_i\,\mathcal{A}}{\rho_i\,S_i\,C_{p_i}}, \quad h_e = \dfrac{U_e\,\mathcal{A}}{\rho_e\,S_e\,C_{p_e}}.$

T_i and T_e are the temperatures of the internal and external fluids, respectively, h_i and h_e are the heat transfer coefficients, v_i and v_e are the velocities, U_i and U_e are the overall heat transfer coefficients, $\mathcal{A}$ is the surface area devoted to heat transfer.

Each PDE requires an initial condition and a boundary condition to be fully defined. The studied heat exchanger is of co-current type. For Eq. (1) describing the temperature of the internal fluid, the boundary condition is usually specified at $z = 0$ as the temperature of the fluid entering the tube is in general known and measurable. Thus, at $z = 0$, it gives

$$T_i(0,t) = T_{i_0}(t) \qquad (3)$$

and most often the initial condition is some given temperature profile at $t = 0$

$$T_i(z,0) = T_i^*(z) \qquad (4)$$

Similarly, for Eq. (2), describing the distribution of temperature of the external fluid in the jacket, the boundary condition is the temperature of the entering fluid T_{e_0}, specified at $z = 0$, consequently

$$T_e(0,t) = T_{e_0}(t) \qquad (5)$$

while the initial condition is some given temperature profile at $t = 0$

$$T_e(z,0) = T_e^*(z) \qquad (6)$$

Eqs. (1)-(6) constitute the dynamic model of the co-current heat exchanger.

3. CONTROL OF THE CO-CURRENT HEAT EXCHANGER

3.1 Control problem formulation

As indicated above, to control the outlet internal temperature T_{i_L}, two manipulated variables are possible, either the inlet external fluid temperature T_{e_0} or the flow rate represented by the velocity v_e. In this work, the temperature T_{e_0}, corresponding to the boundary condition (5), is taken as a manipulated variable to easily control the outlet internal fluid temperature T_{i_L} since the hydrodynamic regime remains invariable. Now, due to Eq. (2), it is noticeable that by manipulating the boundary condition of the jacket, given by Eq. (5), a variation of the temperature of the external fluid T_e along the jacket results. Thus, by denoting as u the control variable and y the controlled variable, the model of the heat exchanger (1)-(6) takes the following form

$$\frac{\partial T_i(z,t)}{\partial t} = -v_i \frac{\partial T_i(z,t)}{\partial z} + h_i \left[T_e(z,t) - T_i(z,t)\right] \qquad (7)$$

$$\frac{\partial T_e(z,t)}{\partial t} = -v_e \frac{\partial T_e(z,t)}{\partial z} + h_e \left[T_i(z,t) - T_e(z,t)\right] \qquad (8)$$

$$T_i(0,t) = T_{i_0}(t) \qquad (9)$$

$$T_e(0,t) = T_{e_0}(t) = u(t) \qquad (10)$$

$$T_i(z,0) = T_i^*(z) \qquad (11)$$

$$T_e(z,0) = T_e^*(z) \qquad (12)$$

$$y(t) = \mathcal{C}\big(T_i(z,t)\big) = \int_0^L \delta(z-L)\,T_i(z,t)\,dz \qquad (13)$$

where $\mathcal{C}(\,.\,)$ is a bounded linear operator.