Annual Reviews in Control 000 (2018) 1–20

Contents lists available at ScienceDirect

Annual Reviews in Control

journal homepage: www.elsevier.com/locate/arcontrol

Vision article

A survey on the analysis and control of evolutionary matrix games

James Riehl[a,*], Pouria Ramazi[b], Ming Cao[c,*]

[a] Department of Electrical and Systems Engineering, Washington University in St. Louis, USA
[b] Department of Mathematical and Statistical Sciences, University of Alberta, Canada
[c] Faculty of Science and Engineering, University of Groningen, Netherlands

A R T I C L E I N F O

Article history:
Received 16 February 2018
Revised 11 April 2018
Accepted 24 April 2018
Available online xxx

Keywords:
Evolutionary games
Population dynamics
Equilibrium convergence
Control strategies

A B S T R A C T

In support of the growing interest in how to efficiently influence complex systems of interacting self-interested agents, we present this review of fundamental concepts, emerging research, and open problems related to the analysis and control of evolutionary matrix games, with particular emphasis on applications in social, economic, and biological networks.

Contents

* Corresponding authors.
E-mail addresses: jrriehl@wustl.edu (J. Riehl), m.cao@rug.nl (M. Cao).

https://doi.org/10.1016/j.arcontrol.2018.04.010

1. Introduction

Whether humans in a community, ants in a colony, or neurons in a brain, simple decisions or actions by interacting individuals can lead to complex and unpredictable outcomes in a population. The study of such systems typically presents a choice between micro- and macro-scale analysis. While there exist intricate micro-models of human decision processes, ant behaviors, and single neurons, assembling these high-dimensional components on a large scale most often results in models that are impenetrable to analysis, and therefore unlikely to reveal any useful properties of the collective dynamics. On the other hand, research on these systems at a broader scale, perhaps subject to substantial simplification of the agent-level dynamics, can help to characterize critical properties such as convergence, stability, controllability, robustness, and performance (Sandholm, 2010). This helps to explain the recent and remarkable trend towards network-based analysis across various disciplines in engineering and the biological and social sciences, which has led to several important discoveries related to system dynamics on complex networks (Chen, Liu, Belabbas, Xu, & Başar, 2017; Madeo & Mocenni, 2015; Olfati-Saber & Murray, 2004; Yu, DeLellis, Chen, Di Bernardo, & Kurths, 2012). For control scientists and engineers, these results facilitate the study of timely and challenging issues related to social, economic, and biological sciences from a control-theoretic perspective.

Evolutionary game theory has emerged as a vital toolset in the investigation of these topics. Originally proposed as a framework to study behaviors such as ritualized fighting in animals (Smith & Price, 1973), it has since been widely adopted in various disciplines outside of biology. The primary innovation of evolutionary game theory is that rather than assuming high levels of rationality in individual choices, perhaps a questionable assumption even for humans, strategies and behaviors propagate through populations via dynamic processes. In the biological world, this propagation is manifested through *survival of the fittest* and reproductive processes, which are widely modeled using *population dynamics* (Ramazi, Cao, & Weissing, 2016; Sandholm, 2010; Weibull, 1997). Systems of first-order differential equations such as *replicator dynamics (RD)* provide an elegant and powerful means to investigate collective behaviors, assuming infinite and well-mixed populations. While these assumptions can lead to reasonable approximations for large, dense populations of organisms, in many other real-world networks, the structure and range of individual interactions plays a major role in the dynamics (Nowak, Tarnita, & Antal, 2010). Fortunately, it is still possible to study replicator-like dynamics in populations connected by networks (Ohtsuki & Nowak, 2006), and it turns out that certain models of imitation reduce exactly to RD in the limit of large networks (Schlag, 1998). Other seemingly more rational decision models such as best-response dynamics (Cortés & Martínez, 2015; Gharehshiran, Krishnamurthy, & Yin, 2017; Gharesifard, Touri, Başar, & Shamma, 2016; Ramazi, Riehl, & Cao, 2016; Shamma & Arslan, 2004) also fit naturally into a network setting, as we will discuss in Section 2.

An extensive literature has emerged in the field of evolutionary games on networks, particularly regarding the question of how cooperation can evolve and persist under various conditions and in various population structures (Axelrod & Hamilton, 1981; Nowak, 2006; Szabó & Fáth, 2007). In this article, rather than survey these works, we will present only some fundamental results in classical evolutionary game dynamics, before discussing some recent developments in the areas of equilibrium convergence and control. Specifically, we set out to achieve three primary goals. First, we aim to introduce the powerful analytical tools of evolutionary game theory to control scientists and engineers not already familiar with the topic. Second, we provide a brief survey of some recent results in the analysis and control of evolutionary matrix games. Third, we discuss some current challenges and open problems in the field for the consideration of interested researchers.

We emphasize that, although game theory and evolutionary game theory are receiving increasing attention as design tools for implementing distributed optimization in industrial and technological systems (Li & Marden, 2013; Quijano et al., 2017; Shamma & Arslan, 2005), including water distribution (Barreiro-Gomez, Ocampo-Martinez, & Quijano, 2017; Ramírez-Llanos & Quijano, 2010), wireless communication (Tembine, Altman, El-Azouzi, & Hayel, 2010), optical networks (Pavel, 2012), and transportation (Altman, El-Azouzi, Hayel, & Tembine, 2009), we focus this survey specifically on the analysis and control of self-organized systems whose constituents are not necessarily subject to design. This stands apart from some ongoing research which aims at engineering the dynamics governing a population of programmable individuals, e.g., robots, in order to drive the state of the system to a desired state. We refer the reader to surveys such as (Marden & Shamma, 2015; Quijano et al., 2017) for detailed discussions on this separate but complementary topic. In contrast, typical individuals we model in this paper such as humans, firms, animals, and neurons are clearly not programmable in the same sense, and even if they were, attempting to do so would likely raise ethical concerns. Rather, for the dynamics of these individuals, we take existing models proposed by biologists, sociologists, and economists,