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Comparative performances of synchronisation between different classes of chaotic systems using three control techniques

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ABSTRACT

This paper puts forward the comparative performances for synchronisation between (i) systems from different chaotic system families, (ii) systems from the Unified Chaotic System (UCS) family, (iii) a hyperchaotic and a chaotic systems and (iv) identical chaotic systems. Three different well-known control techniques, i.e. Nonlinear Active Control (NAC), Sliding Mode Control (SMC) and Adaptive Control (AC) are used for synchronisation between various pairs of chaotic and/or hyperchaotic systems. Performances of NAC, SMC and AC techniques are investigated and compared with synchronisation of different pairs of chaotic systems based on the error dynamics and required control inputs. The integral square error and required control energy measures are considered for comparison. Finally, a generalised view on the use of the control techniques for synchronisation is finally proposed. Moreover, a new chaotic system is proposed and its qualitative analysis is done to illustrate the chaotic behaviour of the system. The new system is used as an example of synchronisation. MATLAB simulation results are presented which reflect the successful achievement of the objectives.

1. Introduction

Synchronisation of chaotic systems has been extensively studied in the last three decades. The study of synchronisation is well identified by the work of Pecora and Carroll in year 1990. In this paper, two chaotic systems with different initial conditions are synchronised. Many chaotic systems and their synchronisation are studied in the variety of applications in the different field of science, engineering and technology such as Biology, Physics, Chemistry, Mathematics, Electrical, Mechanical, Electronics engineering (Chen & Dong, 1998), etc. Recently, synchronisation is being studied in the field of fractional dynamics (Borah & Roy, 2017; Borah, Singh, & Roy, 2016; Shukla & Sharma, 2017a; 2017b; Singh & Roy, 2014; 2016), complex nonlinear systems (Wei, Moroz, Sprott, Akgul, & Zhang, 2017; Wei, Moroz, Sprott, Wang, & Zhang, 2017) for secure communications (Mahmoud, Mahmoud, & Arafa, 2013a; 2013b; 2017). Simultaneously, during 2011, a new classification of chaotic dynamics has been introduced (Leonov, Kuznetsov, & Vagaitsev, 2011; 2012). According to this, there are two types of attractors: self-excitepd attractor and hidden attractor. A self-excitepd attractor has a basin of attraction that is excited from unstable equilibria. In contrast, a hidden attractor has a basin of attraction which does not contain neighborhoods of equilibria. Hidden attractors are important in engineering applications because they allow unexpected and potentially disastropus responses to perturbations in a structure like a bridge or an airplane wing. Recently, findings in the field of hidden attractors about synchronisation (Dudkowski et al., 2016; Jafari, Pham, & Kapitaniak, 2016; Jafari & Sprott, 2013; Jafari, Sprott, & Golpayegani, 2013; Jafari, Sprott, & Nazarimehr, 2015; Leonov et al., 2011; 2012; Pham, Jafari, Volos, & Kapitaniak, 2017; Pham, Vaidyanathan, Volos, & Jafari, 2015; Singh & Roy, 2017a; 2017b; 2017c; Singh, Roy, & Jafari, 2018; Wei, Moroz, Sprott, Akgul, & Zhang, 2017; Wei, Moroz, Sprott, Wang, & Zhang, 2017; Wei, Yu, Zhang, & Yao, 2015; Wei & Zhang, 2014; Wei, Zhang, Wang, & Yao, 2015; Wei, Zhang, & Yao, 2015) is attracting great interest.

Synchronisation between two chaotic systems can be classified in two categories. One is synchronisation between topologically equivalent chaotic systems and other is synchronisation between topologically non-equivalent or different (Hou, Kang, Kong, Chen, & Yan, 2003) chaotic systems. Two chaotic systems are said to be topologically equivalence if there is a homomorphism, mapping orbits of one chaotic system to orbits of other chaotic system homomorphically and preserving orientation of the orbits, otherwise topologically non-equivalent. Synchronisation between various topologically equivalent chaotic systems using NAC, SMC and AC techniques has been studied in the literature

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Table 1
Literature review on different control techniques.

Nonlinear active control	Sliding mode control	Adaptive control
Rossler and Chen chaotic systems (Agiza & Yassen, 2001)	Duffing and Van der Pol oscillators (Salarieh & Alasty, 2009)	hyperchaotic Chen systems (Park, 2005)
The novel and Lorenz chaotic systems (Chen, 2005)	Chen and Lorenz chaotic systems (Liu, 2011)	hyperchaotic Lu systems (Elabbasy et al., 2006)
Duffing system and chaotic pendulum (Lei et al., 2006)	Lorenz-Chen, Chen-Lorenz, Liu-Lorenz chaotic systems (Pourmahmood et al., 2011)	novel hyperchaotic systems derived from the Chen system (Gao et al., 2007)
Li and Cai systems (Sundarapandian, 2011b)	Lu and Genesio-Tesi systems (Jawaada et al., 2012a)	novel chaotic systems derived from the Rossler system (Zhou et al., 2008)
Lorenz and Pehlivan systems (Sundarapandian, 2011c)	Lorenz and Chen systems (Jawaada, Noorani, & Al-sawalha, 2012b)	unified chaotic systems (Yu, 2008)
Genesio and Nuclear spin generator systems (Khan, 2012b)	Qi and Liu chaotic systems (Sundarapandian & Sampath, 2012)	novel chaotic systems derived from Lorenz family (Li et al., 2010)
Windmi and Coullet chaotic systems (Sundarapandian & Rasappan, 2013)	Wang-Chen chaotic systems (Sundarapandian, 2012b)	hyperchaotic Rossler systems (Dimassi & Loria, 2011)
Lu and Rossler systems (Emadzadeh & Haeri, 2005)	Lorenz and Chen systems (Jawaada et al., 2012b)	Newton-Leipnik systems (Khan, 2012a)
Lorenz and Lu systems, Lorenz and Chen systems (Li & Zhou, 2007)	Qi and Liu chaotic systems (Sundarapandian & Sampath, 2012)	novel chaotic systems derived from Lorenz family (El-Dessoky & Yassen, 2012)
Genesio and Rossler systems (Li et al., 2008)	Wang-Chen chaotic systems (Sundarapandian, 2012b)	Lorenz and Chen systems (Zhang et al., 2006)
Lu and Lorenz systems (Al-sawalha & Noorani, 2009b)		Lorenz-Stenflo and Chen systems (Huang, 2008)
hyperchaotic Chen and Lu systems (Al-sawalha & Noorani, 2009a)		Two different novel hyperchaotic systems (Zhu, 2009)
hyperchaotic Liu and hyperchaotic Qi systems (Sundarapandian, 2011a)		hyperchaotic Chen and Second- Harmonic Generator (SHG) (Wu & Zhang, 2009)
hyperchaotic Lorenz and hyperchaotic Lu systems (Al-sawalha & Al-Dababseh, 2011)		Lu and Lorenz-Stenflo hyperchaotic systems (Olusola, 2012)
hyperchaotic Bao and hyperchaotic Xu systems (Sundarapandian, 2012a)		Genesio-Tesi and Li, Li and Lorenz chaotic systems (Srivastava et al., 2013)
hyperchaotic Lorenz and hyperchaotic Chen systems (Sundarapandian & Karthikeyan, 2011)		
Arneodo and Rossler systems (Sundarapandian & Rasappan, 2012),		

(Agiza & Yassen, 2001; Al-sawalha & Al-Dababseh, 2011; Al-sawalha & Noorani, 2009a; 2009b; Dimassi & Loria, 2011; Elabbasy, Agiza, & El-Dessoky, 2006; Gao, Chen, Yuan, & Yu, 2007; Huang, 2008; Jawaada, Noorani, & Al-sawalha, 2012a; 2012b; Khan, 2012b; Lei, Xu, Shen, & Fang, 2006; Li, Chen, & Zhiping, 2008; Li, Leung, Liu, Han, & Chu, 2010; Liu, 2011; Olusola, 2012; Park, 2005; Pourmahmood, Khanmohammadi, & Alizadeh, 2011; Salarieh & Alasty, 2009; Srivastava, Agrawal, & Das, 2013; Sundarapandian, 2011a; 2011b; 2011c; 2012a; 2012b; Sundarapandian & Karthikeyan, 2011; Sundarapandian & Rasappan, 2012; 2013; Sundarapandian & Sampath, 2012; Wu & Zhang, 2009; Yu, 2008; Zhang, Huang, Wang & Chai, 2006; Zhou, Wu, Li, & Xue, 2008; Zhu, 2009). Therefore, synchronisation of topologically different chaotic systems is considered in this paper.

Synchronisation between many pairs of chaotic systems is studied in the literature, where pairs of the chaotic systems are from the UCS family. The literature survey for the synchronisation using nonlinear active control (NAC), sliding mode control (SMC) and adaptive control (AC) is summarised in Table 1. Synchronisation of two chaotic systems is mostly reported using either of these three control techniques. The justification of choosing a control technique is not discussed in these papers.

Some recent findings as in the paper (Cao, Ho, & Yang, 2009), projective synchronisation of a Lorenz chaotic system family is investigated. The drive and response systems are synchronised within a desired scaling factor using an impulsive control technique. The stability analysis of the impulsive functional differential equations derives sufficient conditions. The synchronisation phenomenon in a network model is studied in Cheng and Cao (2011). A feedback control technique is used to achieve synchronisation of the complex networks. Rossler chaotic system is considered at each node of the complex network. The obtained result reveals that the synchronisation is accomplished for growing chaotic network model. The proposed method enhances the synchronizability of the complex model. The paper (Feng & Cao, 2013) investigates the global exponential synchronisation of Chua chaotic systems by designing a novel impulsive controller. The novel impulsive controller, a combination of present and past states of errors, is a modification of the normal impulsive control technique. Global exponential stability criterion is derived for the error system by using impulsive differential equations and differential inequalities. Synchronisation of switched and delay chaotic neural networks with interval parameters uncertainty is investigated in Cao, Alofi, Al-Mazrooei, and Elaiw (2013). Error dynamics is derived based on the theories of the switched systems and drive-response technique. Without constructing Lyapunov–Krasovskii functions, a matrix measure method is used for the first time to switched time-varying delay networks. Synchronisation criteria are derived using Halanay inequality technique for switched interval networks. In the paper (Chen, Cao, Qiu, Alsaedi, & Alsaadi, 2016), synchronisation of multiple chaotic systems with unknown parameters using adaptive control method is proposed. Two different synchronisation modes are considered. One is that more response systems synchronize with one drive system, Lorenz system is drive and two Lu chaotic systems are considered as response systems. Other is the ring transmission synchronisation, which guarantees that all chaotic systems can synchronize with each other. Two hyperchaotic Chen and one hyperchaotic Rossler are considered within the ring structure. Finite-time generalised synchronisation problem of drive-response systems is discussed in Bao and Cao (2016). The finite-time generalised synchro-