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Tutorial Article

A tutorial on modeling and analysis of dynamic social networks. Part II[☆]

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A B S T R A C T

Recent years have witnessed a significant trend towards filling the gap between Social Network Analysis (SNA) and control theory. This trend was enabled by the introduction of new mathematical models describing dynamics of social groups, the development of algorithms and software for data analysis and the tremendous progress in understanding complex networks and multi-agent systems (MAS) dynamics. The aim of this tutorial is to highlight a novel chapter of control theory, dealing with dynamic models of social networks and processes over them, to the attention of the broad research community. In its first part (Proskurnikov & Tempo, 2017), we have considered the most classical models of social dynamics, which have anticipated and to a great extent inspired the recent extensive studies on MAS and complex networks. This paper is the second part of the tutorial, and it is focused on more recent models of social processes that have been developed concurrently with MAS theory. Future perspectives of control in social and techno-social systems are also discussed.

Contents

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1. Introduction

Originating from the early studies on *sociometry* (Moreno, 1934; 1951), Social Network Analysis (SNA) has quickly grown into an interdisciplinary science (Freeman, 2004; Scott, 2000; Scott & Carrington, 2011; Wasserman & Faust, 1994) that has found applications in political sciences (Knoke, 1993; Lazer, 2011), medicine (O'Malley & Marsden, 2008), economics (Easley & Kleinberg, 2010; Jackson, 2008), crime prevention and security (Bichler & Malm, 2015; Masys, 2014). The recent breakthroughs in algorithms and software for big data analysis have made SNA an efficient tool to study online social networks and media (Arnaboldi, Passarella, Conti, & Dunbar, 2015; Kazienko & Chawla, 2015) with millions of users. The development of SNA has inspired many important concepts of modern *network science* (Newman, 2003; Newman, Barabasi, & Watts, 2006; Strogatz, 2001; Van Mieghem, 2006) such as cliques and communities, centrality measures, resilience, graph's density and clustering coefficient.

Employing many mathematical and algorithmic tools, SNA has however benefited little from the recent progress in systems and control (Annaswamy et al., 2017; Murray, 2003; Samad & Annaswamy, 2011). The realm of social sciences has remained almost untouched by control theory, despite the long-term studies on social group dynamics (Diani & McAdam, 2003; Lewin, 1947; Sorokin, 1947) and "sociocybernetics" (Bailey, 2006; Geyer, 1995; Geyer & van der Zouwen, 2001; Wiener, 1954). This gap between SNA and control can be explained, to a great extent, by the lack of dynamic *models* of social processes and mathematical armamentarium for their analysis. Focusing on topological properties of networks, SNA and network science have paid much less attention to dynamics over them, except for some special processes such as e.g. random walks, branching and queueing processes, percolation and contagion dynamics (Newman et al., 2006; Van Mieghem, 2006).

The recent years have witnessed an important tendency towards filling the gap between SNA and control theory, enabled by the rapid progress in multi-agent systems and dynamic networks. The emerging branch of control theory, studying social processes, is very young and even has no name yet. However, the interest of sociologists to this new area and understanding that "coordination and control of social systems is the foundational problem of sociology" Friedkin (2015) leaves no doubt that it should become a key instrument to examine social networks and dynamics over them. Without aiming to provide a exhaustive survey of "social control theory" at its dawn, this tutorial focuses on the most "mature" results, primarily dealing with mechanisms of *opinion formation* (Acemoglu, Dahleh, Lobel, & Ozdaglar, 2011; Castellano, Fortunato, & Loreto, 2009; Dong, Zhan, Kou, Ding, & Liang, 2018; Friedkin, 2015; Hołyst, Kacperski, & Schweitzer, 2001; Xia, Wang, & Xuan, 2011).

In the first part of this tutorial (Proskurnikov & Tempo, 2017), the most classical models of opinion formation have been discussed that have anticipated and inspired the "boom" in multi-agent and networked control, witnessed by the past decades. This paper is the second part of the tutorial and deals with more recent dynamic models, taking into account effects of time-varying graphs, homophily, negative influence, asynchronous interactions and quantization. The theory of such models and multi-agent control have been developed concurrently, inspiring and reinforcing each other.

Whereas analysis of the classical models addressed in Proskurnikov and Tempo (2017) is mainly based on linear algebra and matrix analysis, the models discussed in this part of the tutorial require more sophisticated and diverse mathematical tools. The page limit makes it impossible to include the detailed proofs of all results discussed in this part of the tutorial; for many of them, we have to omit the proofs or provide only their brief sketches.

The paper is organized as follows. Section 2 introduces preliminary concepts and some notation used throughout the paper. Section 3 considers basic results, concerned with properties of the non-stationary French–DeGroot and Abelson models. In Section 4 we consider *bounded confidence* models, where the interaction graph is *opinion-dependent*. Section 5 is devoted to dynamic models based on asynchronous *gossiping* interactions. Section 6 introduces some models, exploiting the idea of *negative influence*. Section 7 concludes the tutorial.

2. Preliminaries and notation

In this section we introduce some notation; basic concepts regarding opinion formation modeling are also recollected for the reader's convenience.

2.1. Notation

We use $m{:}\ n$ to denote the set $\{m, m+1, \dots, n\}$ (here m, n are integer and $m \le n$). Given a vector $x \in \mathbb{R}^n$, $|x|$ stands for its Euclidean norm $|x| = \sqrt{x^\top x}$.

Each non-negative matrix $A = (a_{ij})_{i,j \in V}$ corresponds to the weighted graph $\mathcal{G}[A] = (V, E[A], A)$, whose arcs represent positive entries of A: $a_{ij} > 0$ if and only if $(j, i) \in E(A)$. Being untypical for graph theory (where the entry $a_{ij} > 0$ encodes the arc (i, j)), this notation is convenient in social dynamics modeling (Proskurnikov & Tempo, 2017) and multi-agent control (Ren & Beard, 2008; Ren & Cao, 2011).

Dealing with algorithms' complexity, we use standard Landau–Knuth notation (Knuth, 1976). Given two positive functions $f(n)$, $g(n)$ of the natural argument n, $g(n) = O(f(n))$ stands for the estimate $|g(n)| \le C|f(n)|$, where C is some constant, and $f(n) = \Omega(g(n))$