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Review article

From vehicular platoons to general networked systems: String stability and related concepts☆

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Networked systems and their control are highly important and appear in a variety of applications, including vehicle platooning and formation control. Especially vehicle platoons have been intensively investigated. An interesting problem that arises in this area is string stability, which broadly speaking means that an input signal amplifies unboundedly as it travels through the vehicle string. However, various, not necessarily equivalent, definitions are commonly used. In this paper, we aim to formalise the notion of string stability and illustrate the importance of those distinctions on simulation examples. A second goal is to extend the definitions to general networked systems.

Contents

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1. Introduction

Networked systems and their control are studied in a variety of fields, such as vehicular platooning (Barooah & Hespanha, 2005; Cook, 2007; Herman, Martinec, Hurák, & Šebek, 2015; Lestas & Vinnicombe, 2007; Levine & Athans, 1966; Martinec, Herman, & Sebek, 2016; Melzer & Kuo, 1971; Middleton & Braslavsky, 2010; Peppard, 1974; Rogge & Aeyels, 2008; Seiler, Pant, & Hedrick, 2004; Swaroop & Hedrick, 1996; 1999), formation control (Fax & Murray, 2004; Yadlapalli, Darbha, & Rajagopal, 2006; Zelazo, Rahmani, Sandhu, & Mesbahi, 2008), and many others. These systems consist of many agents that are performing a common task. While in early stages centralised controllers were studied (Levine & Athans, 1966; Melzer & Kuo, 1971), such controllers become infeasible if the number of agents increases. Hence, distributed and decentralised approaches are investigated, where the agents utilise local information and in some cases information transmitted by other agents (Barooah & Hespanha, 2005; Cook, 2007; Fax & Murray, 2004; Herman, Martinec, Hurák, et al., 2015; Li, Duan, & Chen, 2011; Martinec et al., 2016; Middleton & Braslavsky, 2010; Peppard, 1974; Seiler et al., 2004; Swaroop & Hedrick, 1999; Tonetti & Murray, 2010; 2011; Yadlapalli et al., 2006).

However, in some cases these distributed systems experience undesired properties such as instability, amplification of disturbances within the network, and cascading failures. It is therefore of utmost importance to understand the dynamics and limitations that are imposed on these systems with respect to the information flow as well as the underlying systems. In this work we will mostly concentrate on vehicle platoons. In that application two properties are very important, stability and so called string stability. Especially, the latter appears in several variations. In this review we will collect these variations and present them in a more unified framework. Further, we give a possible extension of this property for general networked systems. While not applicable for all systems, the property becomes critical in areas like traffic management and coordination of vehicles.

1.1. Analysis methods

The methods used to analyse these systems range from classical control theory to spatial-temporal systems (Bamieh, Paganini, & Dahleh, 2002; Knorn, 2012; Knorn, Donaire, Agüero, & Middleton, 2014). Recent works combine control theoretic approaches with graph theory. In that context the agent's behaviour is governed by an individual dynamic system, while the information exchange among the agents is represented as a graph. The behaviour of the system is then closely linked to the Laplacian of the graph and in particular its eigenvalues, such that the study of the Laplacian becomes an integral part in the analysis of the networked system (Barooah & Hespanha, 2005; Herman, Martinec, Hurák, et al., 2015; Li et al., 2011; Tonetti & Murray, 2010; 2011; Yadlapalli et al., 2006; You & Xie, 2013). While some works do not consider this link to graph theory directly, their problem description and to some extent the results can be translated into the same separation of the agents individual dynamics and the graph considering the information exchange, see for example Seiler et al. (2004) and Middleton and Braslavsky (2010).

An even newer approach for the analysis of networked systems utilises the so called wave approach (Herman, Martinec, & Veerman, 2016; Herman, Martinec, Veerman, & Sebek, 2015; Martinec et al., 2016). The idea is to model the state of the system as waves propagating through the graph structure. While this method is mainly used in relation to vehicle platooning, the approach can be extended for other structures (Martinec et al., 2016).

Remark 1.1. A related field of research is that of consensus algorithms, where multiple agents aim to equalise a state variable (Moreau, 2005; Olfati-Saber, Fax, & Murray, 2007; Zelazo, Rahmani, & Mesbahi, 2007). Normally, the dynamics of this state variable is simple, for example its derivative is set directly to a weighted average of the state variables of the neighbouring agents. Nonetheless, the similarities between the two fields allow the use of analogous techniques, such as graph theory.

1.2. Problem specifications

The design method selected depends largely but not only on the problem specification. In this context there are four important choices to consider besides the actual controller design:

1. the control objective
2. the individual agent's dynamic system, heterogeneous vs. homogeneous
3. the dynamic system representation of the agents, linear vs. non-linear
4. the interactions between the agents and the communication structure among the agents

1.2.1. Control objective

The controller objective defines what the networked system should achieve as a unit. While some applications allow the control objective to be freely selected or at least relaxed, there will be systems where this is not a valid option. Also, any relaxation of the control objective will lead to a trade-off between the system properties, such as robustness and stability, *vs.* some performance measures. Due to these reasons it is of utmost importance to investigate the properties for a given control objective, as well as investigate what certain relaxations can achieve.

For example, in the application of vehicle platooning the control objective is given as the spacing policy, which determines the inter-vehicle distance the vehicles should maintain. Two methods are prevalent in the literature, which are a constant distance and a constant time gap, respectively. The selection of this objective has a profound impact on the actual stability and performance properties of the system, but also impacts the efficiency at high speeds.

1.2.2. Individual agent dynamics

Independent of the nature of the dynamic system representation and model, it is important to distinguish two approaches for the dynamic systems of all considered agents:

1. homogeneous agents, *i.e.* the dynamic systems and their controllers of all agents are identical (Barooah & Hespanha, 2005; Cook, 2007; Herman, Martinec, Hurák, et al., 2015; Herman et al., 2016; Herman, Martinec, Veerman, et al., 2015; Martinec et al., 2016; Seiler et al., 2004; Tonetti & Murray, 2010; Yadlapalli et al., 2006);
2. heterogeneous agents, *i.e.* the dynamic systems and/or their controllers vary among the agents (Dunbar & Murray, 2006; Lestas & Vinnicombe, 2007; Middleton & Braslavsky, 2010; Tonetti & Murray, 2011).

The use of homogeneous agents simplifies the analysis, however idealises the systems dramatically. Hence, it is important to extend the results where possible to heterogeneous networked systems. This is also relevant in regard to model uncertainties and small