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Review article

Control of shallow waves of two unmixed fluids by backstepping

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A B S T R A C T

Among the existing global challenges, water system management is becoming more and more important as the consumption patterns are continually growing. The implication of water system regulation in irrigated agriculture and production of sustainable energy is self-evident nowadays. In the present paper, new perspectives are given on the control of water flowing in an open channel. Mathematically, these physical processes are described by coupled hyperbolic partial differential equations (PDEs). In view of the recent development in PDE control, backstepping methodology has been proven to be a powerful tool in the sense that it provides a systematic design technique. This paper presents the exponential stabilization results of two shallow wave systems including the shallow waves of two unmixed fluids.

Contents

1. Introduction

The management of water resource involves innumerable environmental and economic challenges of major concern, among which one can mention water management sustainability, intensively irrigated agriculture, flooding phenomena, production of renewable and sustainable energy through hydropower plants. Sev-

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eral efforts have been deployed during the last decades, to represent water management systems as dynamic systems that have the ability to predict consistently water resource evolution over time. From a cohesive perspective, water management systems are complexly integrated, some of which take into account the increasing demand of hydropower that has tremendous implications for the evolution of ecosystems (Winz, Brierley, & Trowsdale, 2009), and may even consist of conflicting sub-systems. For instance, to deal water-related problems that occurs in a complex network of open-channels consisting of

- nodes without storage capacity and nodes with storage capacity such as lakes and reservoirs with infiltration and evaporation,
- channels as river reaches as well as canals, ditches and inter-basin transfers,
- consumptive demands such as irrigated zones or municipal and industrial,

(Andreu, Capilla, & Sanchis 1996) developed a generalized decision-support system (DSS) for water-resources planning and operational management known as AQUATOOL.

The dynamics of open-channel hydraulic systems can be modeled by nonlinear coupled first-order PDEs, derived from the conservation of mass and momentum. For instance, estuaries (Horrevoets, Savenije, Schuurman, & Graas, 2004), rivers (Saint-Venant, 1871), irrigation canals (Malaterre, Rogers, & Schuurmans, 1998), overland flow (Tayfur, Kavvas, Govindaraju, & Storm, 1993; Wang, Chen, Boll, Stockle, & McCool, 2002), lake hydrodynamics (Zhao, Shen, Lai, & III, 1996) as well as coastal circulation (Bouchut, Fernández-Nieto, Mangeney, & Narbona-Reina, 2016; Broche, Salomon, Demaistre, & Devenon, 1986) are described by shallow water dynamic equations also called as *Saint-Venant* equations, neglecting the lateral movement of the water and assuming a constant velocity over the cross-section of an open channel.

The problematic of designing control tools to reinforce the regulation of the water level and the flow rate in open-channel hydraulic systems has a long history and is still driving the attention of researchers due to its challenging aspects. The controllers are usually actuated by adjusting the inflow and the outflow at the two boundaries of the channel. More precisely, changes in the volume of a canal pool connected to an upstream reservoir and a downstream reservoir occur when opening gates are actuated to vary the inflow and the outflow at the two channel boundaries.

Earlier attempts of controller designs consider the approximation of the linearized shallow water equations in the frequency domain as finite-dimensional systems in the spatial coordinate (Corriga, Fanni, Sanna, & Usai, 1982; Corriga, Salimbeni, Sanna, & Usai, 1988; Corriga, Sanna, & Usai, 1983; 1984; Schuurmans, Bosgra, & Brouwer, 1995; Shand, 1971). For example in Corriga et al. (1988), the solutions to the resulting set of ordinary differential equations are given by a distributed transfer matrix relating both the water depth and the water flow discharge at any point in the canal pool to upstream and downstream boundary discharges. Based on these solutions typically given in an analytical closed-loop form, some lumped parameter models equivalent to constant volume control models can then be constructed by accounting for the delay introduced by the wave propagation through two boundaries, enabling the design of simple linear state-feedback controllers. However, all these efforts are based on a non-realistic assumption that the system transfer matrices are uniform with respect to the spatial variable. Indeed, due to the intrinsically nonuniform transfer matrices, such methods are not actually enabling to reduce the complexity of the original control problem. Based on the method of characteristics, proportional boundary feedback controllers are successfully designed to cancel the oscillating modes induced by the reflection of propagating waves on the boundaries of the water pool (Litrico & Fromion, 2006).

Originating from an attempt to deal with a wave equation in Greenberg and Li (1984), more sophisticated controller designs for the shallow water systems are considered, which are based on stability analysis of the distributed parameter models (Coron, de Halleux, Bastin, & Novel, 2002; de Halleux & Bastin, 2002; de Halleux, Prieur, Coron, d'Andréa Novel, & Bastin, 2003; Prieur, Winkin, & Bastin, 2008; Santos & Prieur, 2008). Particularly in Santos and Prieur (2008), a boundary feedback controller is obtained through a direct analysis of the coupled nonlinear *Saint-Venant* equations subject to some perturbations such as frictions. The control performance has been tested successfully using the experimental data of the Sambre river, Belgium and an experimental test bed located in Valence, France. This control framework has then been generalized in Li (1994) for higher order systems.

A major improvement for the stabilization of shallow water equations has been driven by the application of Lyapunov-based control techniques to a one-dimensional *Saint-Venant* model. As stated in Coron, d'Andréa Novel, and Bastin (1999), for a segment, which is of irrigation channel described by *Saint-Venant* equations with two underflow gates at its boundaries, the total energy of the system is not a suitable Lyapunov candidate. Alternatively, the authors constructed an entropy-based Lyapunov function in this same paper, which achieved asymptotic stabilization of the shallow water equations with appropriate upstream and downstream boundary control actions. Since then, systematic Lyapunov-based techniques are used towards achieving efficient controlling of shallow water waves, i.e., the stabilization for coupled systems of one-dimensional hyperbolic PDEs through boundary controllers (Bastin & Coron, 2016). Later on, it was generalized to a "network of systems of conservation laws" in Bastin, Haut, Coron, and d'Andréa Novel (2007) and further improved in Coron, Novel, and Bastin (2007) for systems of conservation laws that can be diagonalized with Riemann Invariants with the introduction of a strict Lyapunov function by choosing properly the boundary control action (see also Tchousso, Besson, & Xu, 2009; Xu & Sallet, 2002 for a class of symmetric linear hyperbolic systems). As a result, Coron et al. (2007) achieved the regulation of the water level and flow in a horizontal open channel, and an extension of the design methodology allows the stabilization of sloping irrigation channels with an arbitrary number of cascading pools (Bastin, Coron, & d'Andréa Novel, 2009).

Various other methods have proven to be effective to ensure stability of such water driven fluvial processes. Some examples are, the proportional-integral boundary feedback controller presented in Santos, Bastin, Coron, and Novel (2008), Xu and Sallet (1999) and Bastin, Coron, and Tamasoiu (2015) (note that a generalization (Xu & Sallet, 1999) for linear hyperbolic systems can be found in Xu and Sallet (2014)), the infinite-dimensional linear matrix inequalities (LMI)-based design proposed in Diagne, Santos, and Rodrigues (2010) and Santos, Rodrigues, and Diagne (2008), and the proportional integral boundary feedback controller in Santos, Wu, and Rodrigues (2014).

Recently, a more complicated shallow water equation involving sediment dynamics has also been investigated. Such dynamics called as *Exner* equation represents the transport of the sediment in a water flow in the case where the sediment moves predominantly as bedload (Bastin & Coron, 2016, Page 25). Exponential stabilization is achieved for coupled linearized *Saint-Venant–Exner* models that are hyperbolic PDE systems by employing various methodologies such as a singular perturbation approach (Tang, Prieur, & Girard, 2014), explicit boundary dissipative conditions (Diagne, Bastin, & Coron, 2012), the ISS-Lyapunov function for time-varying hyperbolic systems (Prieur & Mazenc, 2012), and the backstepping technique (Diagne, Diagne, Tang, & Krstic, 2017). Among these approaches, backstepping is, to the best of our