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Review

Active disturbance rejection control: Old and new results[☆]

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ABSTRACT

The active disturbance rejection control (ADRC), first proposed by Jingqing Han in the 1980s is an unconventional design strategy. It has been acknowledged to be an effective control strategy in the absence of proper models and in the presence of model uncertainty. Its power was originally demonstrated by numerical simulations, and later by many engineering practices. For the theoretical problems, namely, the convergence of the tracking differentiator which extracts the derivative of reference signal; the extended state observer used to estimate not only the state but also the "total disturbance", by the output; and the extended state observer based feedback, progresses have also been made in the last few years from nonlinear lumped parameter systems to distributed parameter systems. The aim of this paper is to review the origin, idea and development of this new control technology from a theoretical perspective. Emphasis will be focused on output feedback stabilization for uncertain systems described by partial differential equations.

Contents

1. Introduction

The capability of dealing with uncertainty is one of the major concerns in modern control theory. There are many well developed control design approaches to cope with uncertainty in control systems. These include the adaptive control for vary or initially uncertain parameters; the internal model principle for regulator problems, the sliding mode control and high gain control for uncertain systems, and robust control which is a paradigm shift in control theory for internal variation and external disturbance. Most of these approaches, however, focus on the worst case scenario which makes the controller rather conservative. The two exceptions are the adaptive control and internal model principle in which the idea of real time estimation/cancelation leads to significant saving of control energy. Let us start with these two approaches to see how and why they are working.

The adaptive control approach was emerged in the 1950s and resurged in the 1970s due to study of uncertain system control in large scale after 1970s (Whitaker, Yamron, and Kezer, 1958. For PDEs, we refer to Krstic, 2010). In the adaptive control approach, the bound of uncertainty is not used and the control varies with

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the uncertainty. Consider feedback stabilization for the following system:

$$\dot{x}(t) = \theta f(x(t)) + u(t), \tag{1.1}$$

where θ is an unknown parameter and $u(t)$ is the control. If we can find an estimator $\hat{\theta}(t)$ for the parameter:

$$\hat{\theta}(t) \to \theta \quad \text{as} \quad t \to \infty, \tag{1.2}$$

then a stabilizing feedback control can be designed as follows:

$$u(t) = -x(t) - \hat{\theta}(t) f(x(t)), \tag{1.3}$$

where the second term in the controller (1.3) is used to cancel the corresponding uncertainty term in (1.1). Substituting (1.3) into (1.1), we can obtain the closed-loop system:

$$\begin{cases} \dot{x}(t) = \tilde{\theta}(t) f(x(t)) - x(t), \\ \tilde{\theta}(t) = \theta - \hat{\theta}(t). \end{cases} \tag{1.4}$$

A Lyapunov function for system (1.4) can be chosen as

$$V(t) = \frac{1}{2}x^2(t) + \frac{1}{2}\tilde{\theta}^2(t).$$

The derivative of $V(t)$ along the solution of (1.4) is found to be

$$\frac{dV(x(t), \tilde{\theta}(t))}{dt} = -x^2(t) + \tilde{\theta}(t)[\dot{\tilde{\theta}}(t) + x(t) f(x(t))] = -x^2(t), \tag{1.5}$$

provided $\dot{\tilde{\theta}}(t) = -x(t) f(x(t))$, and the closed-loop system becomes

$$\begin{cases} \dot{x}(t) = \tilde{\theta}(t) f(x(t)) - x(t), \\ \dot{\tilde{\theta}}(t) = -x(t) f(x(t)). \end{cases} \tag{1.6}$$

Notice that the order of the system is increased by one due to the introduction of the variable $\tilde{\theta}(t)$. By Lasalle's invariance principle and (1.5), it follows that the solution of the system (1.6) satisfies

$$x(t) \to 0 \text{ as } t \to \infty. \tag{1.7}$$

The remaining question is: Is $\hat{\theta}(t) \to \theta (t \to \infty)$? or equivalently $\tilde{\theta}(t) \to 0 (t \to \infty)$? Its answer is not necessarily. Actually, by Lasalle's invariance principle, when $\dot{V}(t) = 0$, we can only conclude that $x(t) = 0$. So $\tilde{\theta} = \tilde{\theta}_0$ may be a nonzero constant satisfying $\tilde{\theta}_0 f(0) = 0$. We therefore have two cases: a) $f(0) \neq 0$ and $\tilde{\theta}_0 = 0$; and b) $f(0) = 0$ and $(x(t), \tilde{\theta}(t)) = (0, \tilde{\theta}_0)$ is a solution of (1.6). The latter case implies that $\tilde{\theta}(t) \to 0 (t \to \infty)$ is not necessarily valid. The former case is just the "persistent exciting" (PE) condition which is $f(0) \neq 0$ for this problem. Nevertheless, in either case, we always have

$$\tilde{\theta}(t) f(x(t)) \to 0 \text{ as } t \to \infty, \tag{1.8}$$

regardless of whether the parameter update law $\dot{\tilde{\theta}}(t) = x(t) f(x(t))$ is convergent or not. In other words, the uncertain term $\theta f(x(t))$ of the system (1.1) is always canceled asymptotically by the feedback control (1.3).

Now we look at the process of internal model principle (IMP) in dealing with external disturbance, which was first introduced in Francis and Wonham (1976) (for PDEs, we refer to Rebarber & Weiss, 2003). Consider once again stabilization for the system:

$$\dot{x}(t) = a(t) + u(t), \tag{1.9}$$

where $u(t)$ is the control and $a(t) = \theta \sin \omega t$ is an external disturbance in which the frequency ω is supposed to be known while the constant amplitude θ is unknown. Since $\ddot{a}(t) = -\omega^2 a(t)$, we can increase the order of system (1.9) as

$$\begin{cases} \dot{x}(t) = a(t) + u(t), \\ \ddot{a}(t) = -\omega^2 a(t), \\ y(t) = x(t), \end{cases} \tag{1.10}$$

where the output of system (1.10) is the state of original system (1.9). Write (1.10) in matrix form:

$$\begin{cases} \dot{X}(t) = AX(t) + Bu(t), \\ y(t) = CX(t), \end{cases} \tag{1.11}$$

where

$$X(t) = (x(t), a(t), \dot{a}(t))^\top, \quad A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega^2 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad C = (1, 0, 0).$$

A simple calculation shows that

$$\text{rank} \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = 3.$$

So system (1.10) or (1.11) is observable. Design the Luenberger observer as

$$\dot{\hat{X}}(t) = A\hat{X}(t) + Bu(t) + L(C\hat{X}(t) - x(t)),$$

where $\hat{X}(t) = (\hat{x}(t), \hat{a}(t), z(t))^\top$, and $L = (\ell_1, \ell_2, \ell_3)^\top$ is selected such that $A + LC$ is Hurwitz. Then we have

$$\begin{cases} \dot{\hat{x}}(t) = \hat{a}(t) + u(t) + \ell_1(\hat{x}(t) - x(t)), \\ \dot{\hat{a}}(t) = z(t) + \ell_2(\hat{x}(t) - x(t)), \\ \dot{z}(t) = -\omega^2 \hat{a}(t) + \ell_3(\hat{x}(t) - x(t)). \end{cases} \tag{1.12}$$

In system (1.10), both $a(t)$ and $\dot{a}(t)$ are regarded as extra state variables. The stabilizing feedback control can thus be designed as

$$u(t) = -\hat{a}(t) - x(t), \tag{1.13}$$

where the first term is used to cancel the external disturbance. In other words, as in the case of adaptive control, we also have used the strategy of estimation and cancelation in the IMP approach.

The active disturbance rejection control (ADRC) further systematically developed the estimation and cancelation approach and greatly enhance its power in dealing with uncertainty in systems. We would like to explain this point by considering feedback stabilization of (1.9) again yet in this case,

$$a(t) = f(x(t), d(t), t), \tag{1.14}$$

which can be used to models (combination of) unknown time-varying, state-dependent internal uncertainty, and external disturbance. The term $a(t)$ is referred to as "total disturbance" in ADRC. The key idea is that regardless of the composition nature of the matter what $a(t)$ is, it is considered as a signal of time and is reflected in the measured output of system. We write system (1.9) as

$$\begin{cases} \dot{x}(t) = a(t) + u(t), \\ \dot{a}(t) = \dot{a}(t), \\ y(t) = x(t), \end{cases} \tag{1.15}$$

where $y(t)$ is the output of extended system (1.15). The exact observerbility of system (1.15) is a trivial problem because if $(y(t), u(t)) \equiv 0$, $t \in [0, T]$, then $a(t) = 0, t \in [0, T]$ and $x(0) = 0$ (Cheng, Hu, and Shen, 2010, p.5, Definition 1.2) for any $T > 0$. This means that $y(t)$ contains all information of $a(t)$. Then a natural idea is: if we can estimate $a(t)$ from $y(t)$ to obtain $\hat{a}(t) \approx a(t)$, then we can also cancel the $a(t)$ in the feedback-loop $u(t) = -\hat{a}(t) + u_0(t)$ where $u_0(t)$ is a new control. Consequently, system (1.15) can be approximated as

$$\dot{x}(t) = u_0(t), \tag{1.16}$$

which is a linear time-invariant system and we have therefore many methods to deal with it. Now, the problem is how the estimation of the total disturbance can be achieved: "$\hat{a}(t) \approx a(t)$".