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Full Length Article

Perspectives in modeling for control of power networks

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A B S T R A C T

Increasing integration of renewable energy sources in the power grid leads to a complete re-thinking of its operation. This necessitates the consideration of new modeling and analysis approaches, which can serve as natural starting points for robust and scalable control and design strategies.

In this 'vision' paper we will highlight two topics within the broad area of power networks: the modeling and analysis of the synchronous generator, and the modeling and analysis of power networks using the swing equation as an approximate model for the generator. In both cases we will discuss a port-Hamiltonian formulation, which reflects the underlying physics of power flow and energy storage. Although the port-Hamiltonian model of the synchronous generator reveals a clear structure it still poses fundamental challenges for its non-zero steady state stability analysis. It is shown how the swing equation can be directly deduced from the power balance of the port-Hamiltonian model of the synchronous generator. Under the phasor assumption, this leads to a port-Hamiltonian model of power networks of generators, which enables a straightforward stability analysis and provides a starting point for control.

1. Introduction

The study of power networks has recently gained much attention from different angles, including the systems and control point of view. Fundamental reasons for the growing interest are the increasing share of *renewables*, such as wind and solar energy, in the electricity production and resulting fundamental challenges in the operation of the network, and at the same time the availability of advanced information technology. This has led, or will lead in the near future, to a re-thinking of the operation of the power grid, replacing the classical top-down structure of energy distribution, from a small number of major power plants to consumers, by a much more horizontal and distributed structure. In such a distributed structure many of the consumers are partially or temporarily producers of energy as well (called *prosumers*). The matching of supply and demand in the resulting complex multi-player setting, subject to a large degree of uncertainty in the energy production of renewables, leads to an operation of the power grid that is increasingly often near its capacity constraints and necessitates the use of advanced communication and control techniques ('*smart grids*'). Another important aspect in the restructuring of the operation of the power grid is the concept of a *micro-grid*. A micro-grid is generally understood as a small-scale power network that is able to operate independently of the main (high-voltage) grid, and therefore is less vulnerable to faults and power outages elsewhere.

In the light of these developments it seems relevant to fundamentally reconsider the basic modeling approaches to power networks, and to investigate how more accurate models reflecting the underlying physics, as well as improved analysis and control concepts, can contribute to novel robust and scalable control methodologies. Indeed, while up to now for many of the control problems in the operation of the power network fairly simple models or linearizations of more involved models could be satisfactory, this is not the case anymore for the control challenges that are posed by the new ways of operating the power grid. In addition, also the classical problem of the stability of the large-scale power network after major disruptions (faults) is still largely open.

Instead of providing a general vision on the area, we restrict ourselves in this introductory and tutorial paper to two topics within the modeling and analysis of power networks. The first topic concerns the detailed modeling for control of the *synchronous generator*, which is (still) the main working horse of the large power grid. The synchronous generator constitutes a fascinating multi-physics system which can be modeled from first principles resulting in a classical eight-dimensional model. We will describe the main characteristics and the mathematical structure of this

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model by utilizing the *port-Hamiltonian* framework. This framework is ideally suited for multi-physics systems and emphasizes aspects of power flow and energy storage, and therefore provides a natural setting for the analysis and control of the dynamics of (networks of) synchronous generators. Although the structure of the port-Hamiltonian formulation of the classical eight-dimensional model is very clear, the analysis and control of this model poses fundamental challenges. Indeed, while the Hamiltonian (total energy) is a natural Lyapunov function for stability of the zero equilibrium, the nonlinearity of the model makes it intrinsically hard to study the stability of steady state values corresponding to non-zero mechanical torque (the interesting case in applications). In this paper we will discuss some of the available analysis methods, and end with a number of numerical simulation studies that motivate further research.

The second topic of this paper concerns the modeling of power networks under the assumption of pure sinusoidal currents and voltages of nominal frequency. Under this assumption the currents and voltages can be compactly represented by their *phasors*, while the network of linear transmission lines linking the generators and the loads can be modeled by an admittance matrix. A trait-d'union between the phasor representation of the transmission line network and the eight-dimensional model of the synchronous generator can be established by a crude approximation of the eight-dimensional model by means of the *swing equation*. We present a simple reasoning leading to the swing equation based on the power-balance of the port-Hamiltonian model of the eight-dimensional model, which we believe to be fairly novel. It turns out that the resulting system of generators and loads linked by the network of transmission lines again defines a port-Hamiltonian system, which admits a straightforward steady state stability analysis. In particular, the model enjoys the property of *shifted passivity* with respect to steady state values, which is an ideal starting point for robust control.

Disclaimer: This paper is not meant to be an introductory survey to modeling and control of power networks, neither a balanced view on new research directions. Instead it focusses on some of our personal (and limited) understandings and tastes in this area, and we are sure that others will write about their own.

2. Port-Hamiltonian modeling of the synchronous generator

An essential component of the alternating-current power grid is the *synchronous generator*; see e.g. Kundur (1993), Machowski, Bialek, and Bumby (2008), and Sauer and Pai (1998). Although the contribution of renewables such as solar and wind energy is growing, the synchronous generators still are the main working horses of the large power grid. While less dominant, they are also important in micro-grids.

The standard model for the synchronous generator, as described e.g. in Kundur (1993), is based on first principles physical modeling. As such it can be naturally written into port-Hamiltonian form given as, see Fiaz, Zonetti, Ortega, Scherpen, and van der Schaft (2013) for details,

$$\begin{bmatrix} \dot{\psi}_s \\ \dot{\psi}_r \\ \dot{p} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -R_s & 0_{33} & 0_{31} & 0_{31} \\ 0_{33} & -R_r & 0_{31} & 0_{31} \\ 0_{13} & 0_{13} & -d & -1 \\ 0_{13} & 0_{13} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \psi_s} \\ \frac{\partial H}{\partial \psi_r} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial \theta} \end{bmatrix} + \begin{bmatrix} I_3 & 0_{31} & 0_{31} \\ 0_{33} & e_1 & 0_{31} \\ 0_{13} & 0 & 1 \\ 0_{13} & 0 & 0 \end{bmatrix} \begin{bmatrix} V_s \\ V_f \\ \tau \end{bmatrix}$$

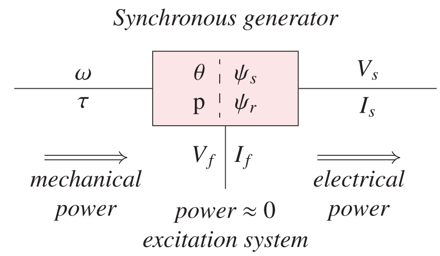

Fig. 1. The state and port variables of the synchronous generator.

$$\begin{bmatrix} I_s \\ I_f \\ \omega \end{bmatrix} = \begin{bmatrix} I_3 & 0_{33} & 0_{31} & 0_{31} \\ 0_{13} & e_1^T & 0 & 0 \\ 0_{13} & 0_{13} & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \psi_s} \\ \frac{\partial H}{\partial \psi_r} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial \theta} \end{bmatrix}, \tag{1}$$

where 0_{lk} denotes the $l \times k$ zero matrix, I_3 denotes the 3×3 identity matrix, and e_1 is the first basis vector of $\mathbb{R}^3$. Eq. (1) is in port-Hamiltonian input-state-output form (van der Schaft, 1996; van der Schaft & Jeltsema, 2014; van der Schaft & Maschke, 1995)

$$\begin{aligned} \dot{x} &= (J(x) - R(x))\frac{\partial H}{\partial x}(x) + g(x)u, \\ y &= g^T(x)\frac{\partial H}{\partial x}(x), \\ J(x) &= -J^T(x), \quad R(x) = R^T(x) \geq 0, \end{aligned} \tag{2}$$

with a *Poisson structure* matrix $J(x)$ given by the constant matrix

$$J = \begin{bmatrix} 0_{66} & & 0_{62} & \\ & & 0 & -1 \\ 0_{26} & & 1 & 0 \end{bmatrix}, \tag{3}$$

and a *resistive structure* matrix $R(x)$, which is also constant, having diagonal blocks

$$R_s = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix}, \quad R_r = \begin{bmatrix} r_f & 0 & 0 \\ 0 & r_{kd} & 0 \\ 0 & 0 & r_{kq} \end{bmatrix}, \quad d, \quad 0, \tag{4}$$

denoting, respectively, the *stator resistances, rotor resistances* and *mechanical friction*. Here $\frac{\partial H}{\partial x}(x)$ throughout denotes the column vector of partial derivatives of the function H.

The state variables x of the eight-dimensional model of the synchronous generator comprise of

- $\psi_s \in \mathbb{R}^3$, the *stator fluxes*,
- $\psi_r \in \mathbb{R}^3$, the *rotor fluxes*: the first one corresponding to the *field winding* and the remaining two to the *damper windings*,
- p, the *angular momentum* of the rotor,
- θ, the *angle* of the rotor.

Moreover, $V_s \in \mathbb{R}^3, I_s \in \mathbb{R}^3$ are the three-phase[1] *stator terminal voltages and currents*, V_f, I_f are the rotor *field winding voltage and current*, and τ, ω are the mechanical *torque* and *angular velocity*; see Fig. 1 for a schematic view.

The *Hamiltonian* H (total stored energy of the synchronous generator) is the sum of the magnetic energy of the generator and the kinetic energy of the rotating rotor, given as the sum of the two nonnegative terms:

$$\begin{aligned} H(\psi_s, \psi_r, p, \theta) &= \frac{1}{2}\begin{bmatrix} \psi_s^T & \psi_r^T \end{bmatrix} L^{-1}(\theta) \begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} + \frac{1}{2J_r}p^2 \\ &= \text{magnetic energy } H_m + \text{ kinetic energy } H_k, \end{aligned} \tag{5}$$

[1] Note that the vectors V_s, I_s are not assumed to be *balanced* (sum zero); see also (21) in the next section.