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Full Length Article

Robust and stochastic model predictive control: Are we going in the right direction?

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ABSTRACT

Motivated by requirements in the process industries, the largest user of model predictive control, we re-examine some features of recent research on this topic. We suggest that some proposals are too complex and computationally demanding for application in this area and make some tentative proposals for research on robust and stochastic model predictive control to aid applicability

1. Introduction

My purpose in this paper is not to present some new theory or procedure; rather my aim is to discuss some difficulties or obstacles that impede the successful application of Model Predictive Control. These difficulties are both theoretical and practical. Our subject now has an excellent foundation created by many researchers. This foundation is not threatened. However, in my opinion, some research does not address industrial needs sufficiently well and there are some topics for which more research is needed. Using a recent review (Mayne, 2014) and a recent paper on model predictive control in industry (Forbes, Patwardham, Hamadah, & Gopulani, 2015) some areas of current research that need further attention or redirection are described. Main attention is given to robust and stochastic model predictive control because these forms of control often require the on-line solution of complex optimal control problems.

The main focus of his paper is on applications in the process industry because the size of the problems routinely considered there makes it difficult, if not impossible, to implement some control algorithms currently proposed for robust and/or stochastic model predictive control. Each installation in the process industries differs, at least in some respects, from its predecessors and has to be separately commissioned. Thereafter it is usually left in the control of an operator who maintains the plant and who needs to understand the plant and its controller. The situation is very different in other areas. As pointed out in Di Cairano (2012), in the automotive industry 'each control design is employed in hundreds or even thousands of final products' referred to as 'large volumes application domains'. The systems in this domain typically have a much smaller state dimension, are much faster, the resultant controllers are unsupervised, have to be much cheaper, and considerably more expert effort can be devoted to each control design making some of the criticisms of current research levelled in this paper irrelevant.

2. Background

The system to be controlled is usually described by

$$x^+ = f(x, u) \tag{1}$$

if there is no disturbance or by

$$x^+ = f(x, u, w) \tag{2}$$

if a disturbance w is present. The state $x \in \mathbb{R}^n$, the control $u \in \mathbb{R}^r$ and the disturbance $w \in \mathbb{R}^p$; it is assumed in the sequel that $E(w) = 0$. Model uncertainty is described in the usual way by

$$x^+ = f(x, u, w)$$
$$y = h(x) \tag{3}$$
$$w = \Delta(\mathbf{y}_t(\cdot)) \tag{4}$$

Δ is a causal input-output operator representing the unmodelled dynamics with input $y(\cdot)$ and output *w;* Δ does not necessarily have a finite-dimensional state representation.

The output $y \in \mathbb{R}^s$ and Δ is an operator representing the unmodelled dynamics that, at time t, maps the output sequence $\mathbf{y}_t \triangleq \{\dots, y(-2), y(-1), y(0), y(1), \dots, y(t)\}$ into $w(t)$. The system is usually subject to some constraints, i.e. the control u is required to lie in a compact set $\mathbb{U} \subset \mathbb{R}^m$ and the state may be required to lie in a closed set $\mathbb{X} \subset \mathbb{R}^n$. The equilibrium (target) state-control pair $(\bar{x}, \bar{u})$ is required to be such that $(\bar{x}, \bar{u})$ lies in the interior of $\mathbb{X} \times \mathbb{U}$. In addition the state x is required to lie in the closed set $\mathbb{X} \subset \mathbb{R}^n$. The finite horizon optimal control problem $\mathbb{P}_N(x)$ solved on-line (N is the horizon) may require the terminal state to lie in the compact

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set $X_f \subset \mathbb{R}^n$; this is a constraint on the optimal control problem and is *not* a system constraint. In robust model predictive control it is assumed that the disturbance w takes values in the compact set $\mathbb{W} \subset \mathbb{R}^p$ that contains the origin in its interior. In stochastic model predictive control $\{w(t)\}$ is a random process, a sequence of independent, identically distributed random variables taking values in a set $\mathbb{W} \subset \mathbb{R}^p$ that is not necessarily compact. In the stochastic case it is assumed that there is an underlying probability space with probability measure P.

The decision variable for the optimal control problem varies considerably. In conventional model predictive control in which the system is described by (1), the decision variable is the control sequence $\mathbf{u} = \{u(0), u(1), \ldots, u(N-1)\} \in \mathbb{R}^{Nm}$; this is one of the big attractions of model predictive control since off-line determination of a control law $\kappa : \mathbb{R}^n \to \mathbb{R}^m$, a complex task, is replaced by on-line determination of a control sequence $\mathbf{u}$ for each encountered value of the state x. The decision variable $\mathbf{u}$ is also employed fairly often in the literature on robust on stochastic model predictive control. In order to overcome the disadvantages, discussed below, of using $\mathbf{u}$ as a decision variable for robust or stochastic model predictive control, a feedback policy $\pi \triangleq \{\mu_0(\cdot), \mu_1(\cdot), \ldots, \mu_{N-1}(\cdot)\}$, a sequence of measurable control laws, is also employed; for each i, $\mu_i : \mathbb{R}^n \to \mathbb{R}^m$. Optimizing over arbitrary functions is too complex so π is often parameterized by a vector $v = (v_0, v_1, \ldots, v_{N-1})$ with $\mu_i(x) \triangleq \theta(x, v_i)$; e.g. $\theta(x, v_i) = \sum_{j\in J} v_i^j \phi_j(x)$ in which $\{\phi_j(\cdot) | j \in J\}$ is a set of pre-specified functions. When the system $f(\cdot)$ is linear, a common choice is $\mu_i(x) = \theta(x, v_i) = v_i + Kx$, K chosen so that $A + BK$ is stable, a parameterization suggested by Rossiter, Kouvaritakis, and Rice (1998). The decision variable $\mathbf{u}$ may be regarded as a degenerate policy in which $\mu_i(x) = \theta(x, v_i) = v_i = u_i$ for all i, all x. Let Π denote the class of policies π defined above, i.e. $\Pi \triangleq \{\pi = \{\mu_0(\cdot), \mu_1(\cdot), \ldots, \mu_{N-1}(\cdot) \mid \mu_i(x) = \theta(x, v_i), i = 0, 1, \ldots N-1\}$. Optimizing with respect to $\pi \in \Pi$ is equivalent to optimizing with respect to the vector sequence $\mathbf{v} = \{v_0, v_1, \ldots, v_{N-1}\}$.

2.1. Definition of cost function $V_N(x, \mathbf{u})$ or $V_N(x, \pi)$

For nominal model predictive control, in which the system is assumed to satisfy (1), $x^{\mathbf{u}}(i; x)$ denotes the solution of (1) at time i given that the initial state is x at time 0 and the control is $\mathbf{u}$. For robust or stochastic model predictive control, in which the system is assumed to satisfy (2), $x^{\pi}(j; x, \mathbf{w})$ denotes the solution of

$$x(i+1) = f(x(i), \mu_i(x(i)), w(i)), i = 0, 1, \ldots, N-1 \tag{5}$$

at time j given that the initial state is $x(0) = x$ and the control policy is $\pi = \{\mu_0(\cdot), \mu_1(\cdot), \ldots, \mu_{N-1}(\cdot)\} \in \Pi$. The definition of cost depends on the type of model predictive control: conventional, robust or stochastic:

1. Conventional MPC:

$$V_N(x, \mathbf{u}) \triangleq \sum_{i=0}^{N-1} \ell(x^{\mathbf{u}}(i; x), u(i)) + V_f(x^{\mathbf{u}}(N; x)) \tag{6}$$

2a. Robust MPC – Nominal cost:

$$V_N(x, \pi) \triangleq \sum_{i=0}^{N-1} \ell(x^{\pi}(i; x, \mathbf{0}), \mu_i(x^{\pi}(i; x, \mathbf{0})) + V_f(x^{\pi}(N; x, \mathbf{0})) \tag{7}$$

2b. Robust MPC – Worst case cost:

$$V_N(x, \pi) \triangleq \max_{\mathbf{w}\in\mathbb{W}^N} \sum_{i=0}^{N-1} \ell(x^{\pi}(i; x, \mathbf{w}), \mu_i(x^{\pi}(i; x, \mathbf{w})) + V_f(x^{\pi}(N; x, \mathbf{w})) \tag{8}$$

in which $\mathbf{0} \triangleq \{0, 0, \ldots, 0\}$ is a sequence of zero vectors.

3a. Stochastic MPC – Nominal cost:

$$V_N(x, \pi) \triangleq \sum_{i=0}^{N-1} \ell(x^{\pi}(i; x, \mathbf{0}), \mu_i(x^{\pi}(i; x, \mathbf{0})) + V_f(x^{\pi}(N; x, \mathbf{0}) \tag{9}$$

3b. Stochastic MPC – Expected cost:

$$V_N(x, \pi) \triangleq E_{|x} \sum_{i=0}^{N-1} \ell(x^{\pi}(i; x, \mathbf{w}), \mu_i(x^{\pi}(i; x, \mathbf{w})) + V_f(x^{\pi}(N; x, \mathbf{w})) \tag{10}$$

in which $E_{|x}(\cdot) \triangleq E(\cdot|x)$ and E is expectation under P, the probability measure of the underlying probability space.

2.2. Definition of constraint set $\mathcal{U}_N(x)$ or $\Pi_N(x)$

Constraints also depend on the type of model predictive control that is employed:

1. Nominal MPC: For each x, $\mathcal{U}_N(x)$ is the set permissible control sequences $\mathbf{u}$. Each $\mathbf{u} \in \mathcal{U}_N(x)$ satisfies:

$$u(i) \in \mathbb{U},\ x^{\mathbf{u}}(i; x) \in \mathbb{X}, \forall i \in \mathbb{I}_{0:N-1},\ x^{\mathbf{u}}(N; x) \in X_f \tag{11}$$

It is assumed here and in the sequel that $X_f \subset \mathbb{X}$.

2. Robust MPC: For each x, $\Pi_N(x)$ is the set of permissible control policies. Each $\pi = \{\mu_0(\cdot), \mu_1(\cdot), \ldots, \mu_N(\cdot)\} \in \Pi_N(x)$ satisfies:

$$\mu_i(x^{\pi}(i; x, \mathbf{w})) \in \mathbb{U},\ x^{\pi}(i; x, \mathbf{w}) \in \mathbb{X}, \forall (i, \mathbf{w}) \in \mathbb{I}_{0:N-1} \times \mathbb{W}^N,\ x^{\pi}(N; x, \mathbf{w}) \in X_f, \forall \mathbf{w} \in \mathbb{W}^N \tag{12}$$

in which $\mathbb{I}_{a:b} \triangleq \{a, a+1, \ldots, b-1, b\}$.

3. Stochastic MPC: Because the probability density of the disturbance w does not have finite support, it is impossible to satisfy the state and terminal constraints almost surely. To obtain a meaningful optimal control problem, it is necessary to 'soften' the state and terminal constraints. For process control applications, the control constraint must *always* be satisfied, a requirement sometimes ignored in the literature. Two methods for 'softening' the constraint have been used in the literature. In the first (Primbs & Sung, 2009), 'hard' constraints of the form $x(w) \in \mathbb{X}$ for all $w \in \mathbb{W}$ are replaced by the average constraint $E(x(w)) \in \mathbb{X}$. In the second (Kouvaritakis, Cannon, Raković, & Cheng, 2010; Prnadini, Garatti, & Lygeros, 2012) the constraint $x(w) \in \mathbb{X}$ for all $w \in \mathbb{W}$ is replaced by $P(x(w) \in X) \geq 1 - \varepsilon$ for some $\varepsilon \in (0, 1)$. Hence, the constraints employed in the optimal control problem solved on-line take the form

$$\mu_i(x^{\pi}(i; x, \mathbf{w})) \in \mathbb{U},\ E_{|x}(x^{\pi}(i; x, \mathbf{w})) \in \mathbb{X} \quad \forall i \in \mathbb{I}_{0:N-1},\ E_{|x}(x^{\pi}(N; x, \mathbf{w})) \in X_f)\ \forall \mathbf{w} \in \mathbb{W}^N, \tag{13}$$

in which $E_{|x}(\cdot) \triangleq E((\cdot)|x)$ when average constraints are employed, or

$$\mu_i(x^{\pi}(i; x, \mathbf{w})) \in \mathbb{U},\ P_{|x}(x^{\pi}(i; x, \mathbf{w})) \in \mathbb{X}) \geq 1 - \varepsilon,\ \forall i \in \mathbb{I}_{0:N-1},\ P_{|x}(x^{\pi}(N; x, \mathbf{w})) \in X_f) \geq 1 - \varepsilon\ \forall \mathbf{w} \in \mathbb{W}^N \tag{14}$$

in which $P_{|x}(\cdot) \triangleq P(\cdot|x)$ when probabilistic constraints are employed. Let $\Pi_N(x)$ denote the set of policies $\pi \in \Pi$ satisfying the appropriate constraints, average or probabilistic. The possibility of satisfying the *hard* control constraint, which is necessary in process control applications, is discussed below.

2.3. Definition of nominal optimal control problem $P_N(x)$

For nominal model predictive control, the optimal control problem, $P_N(x)$, that is solved on-line is:

$$P_N(x): \qquad V_N^0(x) = \min_{\mathbf{u}\in\mathcal{U}_N(x)} V_N(x, \mathbf{u}) \tag{15}$$