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Single range observability for cooperative underactuated underwater vehicles

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ABSTRACT

The paper addresses the single range observability analysis of a kinematics model of cooperating underactuated underwater vehicles. Teams of underwater vehicles that communicate with each other may be able to access and exchange their relative distances through, by example, acoustic signal time-of-flight measurements. Such relative distance measurements together with vehicle's attitude and velocity information may be used onboard to implement a navigation filter to estimate the vehicle's relative positions and orientations. A pre-requisite for successfully designing such navigation filters is to assess the systems observability properties. Contrary to the majority of existing studies on single range observability, the paper considers a more realistic underactuated kinematics model for slender body autonomous underwater vehicles rather than a simple point mass model. The paper extends previous results building on an augmented state technique allowing to reformulate the nonlinear observability problem in terms of a linear time varying one. As a result, all possible (globally) unobservable motions are characterized in terms of the systems' initial conditions and velocity commands within the class of interest. The fundamental results reported are also illustrated by numerical simulations providing evidence of different motions generating the same output, namely lacking observability.

1. Introduction

Multi robot systems in air, land and marine applications have received an increasing amount of attention in last years. Indeed many applications as sampling (Antonelli, Chiaverini, and Marino, 2012 and references therein), surveillance, mapping and exploration can benefit in robustness and coverage by exploiting cooperating teams of robots rather than single vehicle systems. In particular, research effort is targeting the issue of designing distributed and cooperative control schemes minimizing the need of a centralized team controller. Distributed motion control architectures for cooperative robots require, in general, that the team members share *some* knowledge about their relative states: typically the relative positions (and eventually velocities) need to be known among neighboring vehicles in order to accomplish cooperative motion. In several applications, as underwater ones where sensors are mostly based on acoustics, team members can measure their relative distances only (Soares, Aguiar, Pascoal, & Gallieri, 2012). This poses a remarkable problem of observability (also known as single beacon navigation in the literature): given a kinematics model of, say, two vehicles, will their relative position and orientation (pose) be observable based on relative Euclidean distance measurement only? As the Euclidean distance is a nonlinear function of the relative position vector, the observability problem is nonlinear even for point mass (linear) kinematics model as in Arrichiello, Antonelli, Aguiar, and Pascoal (2011). Single beacon navigation problems have been addressed in the area of wheeled mobile robotics for relative localization (refer to Martinelli and Siegwart (2005) and Zhou and Roumeliotis (2008), for example). With reference to marine robotics applications, the issue of single beacon navigation (and observability analysis) has been addressed by several authors including Arrichiello et al. (2011), Bahr (2009), Bahr, Leonard, and Fallon (2009), Batista, Silvestre, and Oliveira (2011), Bayat (2015), Bayat, Crasta, Aguiar, and Pascoal (2015), Fallon, Papadopoulos, Leonard, and Patrikalakis (2010), Gadre and Stilwell (2004), Jouffroy and Ross (2005), Jouffroy and Reger (2006), Olson, Leonard, and Teller (2004), Quenzer and Morgansen (2014), Webster, Eustice, Singh, and Whitcomb (2013) and Webster, Whitcomb, and Eustice (2010). These studies focus on simple (point-mass like) kinematic models often in $2D$ only. The observability issues arising in single beacon navigation are similar to the observability properties of tracking systems. Although tracking systems are more often based on bearing only measurements, the problem of tracking through range-only measurements has received some attention also in oceanic engineering applications (Maki, Matsuda, Sakamaki, Ura, & Kojima, 2013; Song, 1999).

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Based on a recent approach to address the *global* observability of a system model made of two underactuated vehicles, this paper extends previous results (Parlangeli & Indiveri, 2014; Parlangeli, Pedone, & Indiveri, 2012) by including the case where both vehicles have constant, but non null, linear and angular velocities. From a methodological point of view, the proposed observability analysis is inspired by the work of Batista, Silvestre, and Oliveira (2010) and Batista et al. (2011) where a similar single range observability issue has been addressed for a different (point-mass) kinematics model.

In Section 2 the system model is illustrated and the observability problem is defined. In Section 3 the adopted observability tools and methods are described whereas the main results of the analysis are reported in Section 4. Simulation results providing numerical evidence of unobservable trajectories are illustrated in Section 5 and conclusions are finally addressed in Section 6. An Appendix section with a few technical results is also included after the Bibliography.

2. System model

2.1. Notation

Vectors are denoted with lower case boldface fonts and matrices with capital roman letters. Reference frames are labeled as <1>, <2>, etc. Given a vector $\mathbf{p} \in \mathbb{R}^3$ its representation in frame <1> will be denoted as ${}^1\mathbf{p}$ having components of $({}^1\mathbf{p})_1, ({}^1\mathbf{p})_2, ({}^1\mathbf{p})_3$. The norm of vector $\mathbf{p}$ will be equivalently indicated with $\|\mathbf{p}\|$ or p. The unit vector of $\mathbf{p} \neq \mathbf{0}$, namely $\mathbf{p}/\|\mathbf{p}\|$, will be indicated with $\check{\mathbf{p}}$. The set of all unit vectors in $\mathbb{R}^3$ will be denoted by $\mathbb{S}^2$. The special orthogonal group of 3*D* rotation matrices is $SO(3)$ and the rotation matrix between frames <2> and <1> will be indicated with ${}^1R_2$ such that ${}^1\mathbf{p} = {}^1R_2\,{}^2\mathbf{p}$. The 3*D* skew symmetric matrix associated to vector product will be indicated with $S(\mathbf{a})$, namely for any $\mathbf{a} = (a_1, a_2, a_3)^\top \in \mathbb{R}^3$

$$S(\mathbf{a}) := \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \in \mathbb{R}^{3\times 3} \tag{1}$$

such that $S(\mathbf{a})\mathbf{b} = \mathbf{a} \times \mathbf{b}$. A dot on a variable (either a scalar a vector or a matrix) indicates its time derivative. The angular velocity vector associated to the rotation matrix ${}^1R_2$ is defined as the axial vector satisfying

$${}^1\dot{R}_2\,{}^1R_2^\top = S({}^1\boldsymbol{\omega}_{2/1}) \tag{2}$$

namely ${}^1\boldsymbol{\omega}_{2/1}$ is the angular velocity of frame <2> with respect to frame <1> expressed in frame <1>. The symbol $\otimes$ will be used for the Kronecker product (Laub, 2005) between two matrices, namely given $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ the matrix $A \otimes B \in \mathbb{R}^{mp\times nq}$ is defined as:

$$A \otimes B = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix} \tag{3}$$

and $\oplus$ denotes the Kronecker sum (Laub, 2005) such that for any two square matrices $C \in \mathbb{R}^{n\times n}$ and $D \in \mathbb{R}^{m\times m}$ the matrix $(C \oplus D) \in \mathbb{R}^{mn\times mn}$ is defined as

$$C \oplus D = (I_{m\times m} \otimes C) + (D \otimes I_{n\times n}) \tag{4}$$

where $I_{l\times l} \in \mathbb{R}^{l\times l}$ is the *l*-dimensional identity matrix for any nonnegative integer *l*. The columns of $I_{l\times l}$ will be denoted with $\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_l$. We denote with *j* the imaginary unit. The set of unobservable vectors is denoted with X_{no}.

2.2. Vehicles model

The kinematics model considered is a 3*D* underactuated vehicle (as a torpedo shaped submersible, a missile or airplane) having a linear velocity with an arbitrarily assigned norm and direction along a unit vector (usually the surge direction) that can be rotated with a desired angular velocity. Mathematically this simple model is captured by the following equations:

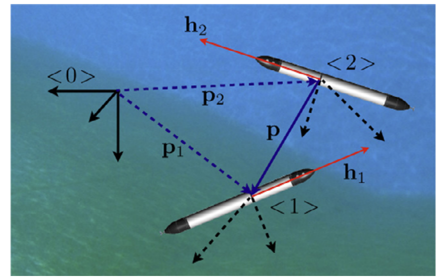

Fig. 1. Geometry of the problem.

$$\dot{\mathbf{q}} = u\,\mathbf{h} \quad : \quad \|\mathbf{h}\| = 1 \tag{5}$$

$$\dot{\mathbf{h}} = \boldsymbol{\omega} \times \mathbf{h} \tag{6}$$

where $\mathbf{q}$ is the position (with respect to an earth-fixed frame) of the origin of a body-fixed frame, $\mathbf{h}$ is the unit vector of its linear velocity having norm $|u|$ and $\boldsymbol{\omega}$ is its angular velocity. Eqs. (5) and (6) define a kinematics control system with state vector $\mathbf{x} = (\mathbf{q}^\top, \mathbf{h}^\top)^\top \in \mathbb{R}^3 \times \mathbb{S}^2$ and inputs $u \in \mathbb{R}$ and $\boldsymbol{\omega} \in \mathbb{R}^3$. This nonlinear model can be viewed as the 3*D* version of the classical planar unicycle nonholonomic model in 2*D*. Notice that while many torpedo shaped autonomous underwater vehicles (AUVs) cannot turn on the spot due to the use of control surfaces (only) for the angular velocity actuation, some AUVs (Caffaz, Caiti, Casalino, & Turetta, 2010) are equipped with side thrusters that allow to actively control pitch and yaw velocities also at zero surge. Indeed the vehicles considered in this paper are of the latter kind (Caffaz et al., 2010), hence the input angular velocity $\boldsymbol{\omega}$ in Eq. (6) will be assumed to be independent of the surge speed *u*. Moreover, although Eq. (6) does not pose constraints on the roll component $\boldsymbol{\omega}^\top\mathbf{h}$ the fact that such component might not be actuated does not limit the generality of the observability analysis developed in the reminder of the paper.

With reference to Fig. 1 consider an earth fixed frame <0> and two body fixed (moving) frames <1> and <2> having origin in $\mathbf{p}_1$ and $\mathbf{p}_2$ respectively. Frames <1> and <2> are assumed to move according to the kinematics equations:

$${}^0\dot{\mathbf{p}}_i(t) = u_i(t)\,{}^0\mathbf{h}_i(t), \quad \|{}^0\mathbf{h}_i(t)\| = 1 \tag{7}$$

$${}^0\dot{\mathbf{h}}_i(t) = {}^0\boldsymbol{\omega}_{i/0}(t) \times {}^0\mathbf{h}_i(t) \tag{8}$$

for $i = 1, 2$. In accordance to the discussion of the model in Eqs. (5) and (6), u_i and $\boldsymbol{\omega}_{i/0}$ are the linear and angular velocities respectively of the two systems and $\mathbf{h}_i$ are two unit vectors. We assume that $u_1(t)$ and $u_2(t)$ cannot be identically zero at the same time. Without loss of generality, in the following it will be assumed that ${}^i\mathbf{h}_i$ is the *x*−axis unit vector of frame <*i*>, namely ${}^i\mathbf{h}_i = {}^i\mathbf{e}_1 = (1, 0, 0)^\top$. Denoting with

$$\mathbf{p} := \mathbf{p}_2 - \mathbf{p}_1 \tag{9}$$

the relative position of frame <2> with respect to <1>, we are interested in analyzing the motion of the vehicle <2> as viewed by the observing vehicle <1> : standard kinematics calculations based on the projection of Eqs. (7) and (8) on frame <1> lead to the following:

$${}^1\dot{\mathbf{p}}(t) = u^1_{2(t)}\mathbf{h}_2(t) - u^1_{1(t)}\mathbf{h}_1(t) - S({}^1\boldsymbol{\omega}_{1/0}(t))\,{}^1\mathbf{p}(t) \tag{10}$$

$${}^1\dot{\mathbf{h}}_1(t) = \mathbf{0} \tag{11}$$

$${}^1\dot{\mathbf{h}}_2(t) = S({}^1\boldsymbol{\omega}_{2/1}(t))\,{}^1\mathbf{h}_2(t). \tag{12}$$