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Local Decomposition and Accessibility of Nonlinear Infinite-Dimensional Systems

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Abstract: This article deals with the local system decomposition of infinite-dimensional systems, which are described by second-order nonlinear partial differential equations. We show that if there exists a certain codistribution which is invariant under the generalized system vector field, a local triangular decomposition can be obtained. Furthermore, we draw connections to a different approach which is based on transformation groups. Throughout the article we apply differential geometric methods, highlighting the geometric picture behind the system description. The article is closed with a nonlinear example.

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1. INTRODUCTION AND MOTIVATION

It is well-known that for finite-dimensional systems described by ordinary differential equations (ode's), local decompositions obtained from certain invariant distributions or codistributions allow to highlight system properties such as accessibility and observability, see e.g. H. Nijmeijer, A. J. van der Schaft (1991). This approach makes use of special coordinate transformations to visualize non-accessible or non-observable subsystems. Hence, one is able to draw conclusions about accessibilty or observability by a structural analysis.

K. Rieger, M. Schöberl, K. Schlacher (2010) demonstrated that the system decomposition by exploiting the existence of an invariant codistribution can be extended to systems described by first-order nonlinear partial differential equations (pde's). In case a system allows such a decomposition, system properties like accessibility and observability can also be examined by a structural analysis. Our contribution extends this approach to systems described by second-order nonlinear pde's. It should be noted that from a triangular decomposition the non-accessibility of the system follows, but in general the converse is not true. Thus, a triangular decomposition only yields a sufficient condition for non-accessibility in the pde-scenario.

In the literature there can be found further approaches for tackling the accessibility or observability problem. For instance, R. F. Curtain, H. J. Zwart (1995) draw conclusions about accessibility or observability by use of methods from the area of functional analysis, i.e. by examining certain maps.

Moreover, a weaker notion of the accessibility or observability problem can for instance be analyzed by a finite-dimensional approximation of the pde's, e.g. by means of a modal approximation as proposed in E. D. Gilles (1973). K. Schlacher, A. Kugi, K. Zehetleitner (2002) showed that transformation groups are an appropriate tool to tackle the accessibility and observability problem for systems described by ode's. K. Rieger, K. Schlacher (2010); K. Rieger (2009) successfully extended the transformation group approach to systems described by pde's. We briefly recapitulate this method, with focus on the systems we discuss in this article. The aim is to link the results of the transformation group approach to the one based on invariant codistributions.

To sum up, in this contribution we are extending an existing system decomposition approach from first to second-order nonlinear pde's. Furthermore, we will make a connection between this approach and a different one based on transformation groups.

1.1 Mathematical Preliminaries

In this article we are using the notation and methods of differential geometry. In particular, we are applying the usual notation of jet calculus and exterior algebra. For a more comprehensive treatise of these topics, we refer to e.g. D. J. Saunders (1989); P. Griffiths, R. Bryant, S. Chern, R. Gardner, H. Goldschmidt (2012). Formulas are kept short by applying Einstein's summation convention, and not indicating the index ranges if they are clear from context. Moreover, to avoid mathematical subtleties we suppose all manifolds to be smooth, and all system functions to depend smoothly on their arguments. Pullback bundles will not be explicitly indicated if they are clear from context, to avoid an exaggerated notation. We use the standard symbol $\wedge$ for the wedge product (exterior product), and $\rfloor$ denotes the natural contraction between tensor fields. In the following, we are using bundle structures to be able to distinguish between dependent and independent coordinates. Throughout the whole article we will use Greek

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letters to indicate the indices of dependent coordinates, and Latin letters for the indices of independent ones.

Let us consider the bundle $\pi : \mathcal{X} \to \mathcal{D}$, where π is a surjective submersion, called projection, from a manifold $\mathcal{X}$ with coordinates $\left(X^i, x^\alpha\right)$ to a manifold $\mathcal{D}$ with coordinates X^i. In this setting x^α are the dependent and X^i the independent coordinates, with the corresponding index ranges $\alpha = 1, \ldots, n$ and $i = 1, \ldots, q$. Here $\mathcal{D}$ is the spatial domain of the considered pde's, which is supposed to be a compact manifold, with a global volume form $\mathrm{VOL} = \mathrm{d}X^1 \wedge \ldots \wedge \mathrm{d}X^q$ and a coherently orientable boundary $\partial\mathcal{D}$. Complementary, $\partial\mathcal{D}$ is endowed with the adapted coordinates $X_\partial = \left(X^i\right)$, with $i = 1, \ldots, q-1$. Furthermore, a map $\phi : \mathcal{D} \to \mathcal{X}$, $x^\alpha = \phi^\alpha(X^i)$ is called a section of the bundle $\pi : \mathcal{X} \to \mathcal{D}$, and $\Gamma(\mathcal{D}, \mathcal{X})$ indicates the set of all sections.

The first and second jet manifold $J^1(\mathcal{X})$, $J^2(\mathcal{X})$ can be introduced endowed with the coordinates $(X^i, x^\alpha, x_i^\alpha)$, resp. $(X^i, x^\alpha, x_i^\alpha, x_{ij}^\alpha)$. The coordinates x_i^α and x_{ij}^α are so-called derivative coordinates (jet variables) of first and second order (derivatives of the dependent coordinates with respect to the independent ones). Moreover, for a section $\phi^\alpha(X^i)$ the following relations regarding the derivative coordinates x_i^α and x_{ij}^α

$$x_i^\alpha \circ \phi = \frac{\partial \phi^\alpha}{\partial X^i}, \quad x_{ij}^\alpha \circ \phi = \frac{\partial^2 \phi^\alpha}{\partial X^i \partial X^j},$$

hold. Partial derivatives with respect to $X^i, x^\alpha, x_i^\alpha, x_{ij}^\alpha$ will be denoted as $\partial_i, \partial_\alpha, \partial_\alpha^i, \partial_\alpha^{ij}$ for short. With the help of the jet manifolds several bundles can be formed. For our purpose the two bundle structures $\pi_0^2 : J^2(\mathcal{X}) \to \mathcal{X}$ and $\pi^1 : J^1(\mathcal{X}) \to \mathcal{D}$ play an important role. Based on the already mentioned bundles, we are able to introduce several tangent bundle structures. We will restrict ourselves to the vertical tangent bundle $\nu_\mathcal{X} : \mathcal{V}(\mathcal{X}) \to \mathcal{X}$, and the cotangent bundle $\tau_\mathcal{X}^* : \mathcal{T}^*(\mathcal{X}) \to \mathcal{X}$. A typical element of the latter one reads as $\omega = \omega_\alpha \mathrm{d}x^\alpha + \omega_i \mathrm{d}X^i$, and is called a one-form. A vertical vector field $v : \mathcal{X} \to \mathcal{V}(\mathcal{X})$ is given in local coordinates by $v = v^\beta(X^i, x^\alpha)\partial_\beta$. Additionally, with $v(\cdot)$ we denote the Lie derivative of a function or a one-form with respect to the vector field v. The first and second jet-prolongation of a vertical vector field read as

$$\begin{aligned} j^1(v) &= v^\beta \partial_\beta + d_i(v^\beta)\partial_\beta^i, \\ j^2(v) &= v^\beta \partial_\beta + d_i(v^\beta)\partial_\beta^i + d_j(d_i\left(v^\beta\right))\partial_\beta^{ij}, \end{aligned}$$

with d_i as the total derivative vector field with respect to X^i. Since there exists a symmetry in the second-order derivative coordinates we have to use a slightly modified version of the summation convention. Thus, we agree on the index range $1 \le i \le j \le q$ for all occurring double indices to avoid multiple appearing equal terms.

To handle second-order pde's we introduce so-called generalized vertical vector fields. These vector fields $v : J^2(\mathcal{X}) \to \pi_0^{2,*}(\mathcal{V}(\mathcal{X}))$ read as $v = v^\beta(X^i, x^\alpha, x_i^\alpha, x_{ij}^\alpha)\partial_\beta$ and can be introduced with the help of a pullback bundle structure [1]. Finally, systems described by pde's will be denoted as pde-systems, and their accompanying boundary conditions as bc's for short.

[1] More precisely, the total space manifold is defined as $\left\{(a,b) \in J^2(\mathcal{X}) \times \mathcal{V}(\mathcal{X}) : \pi_0^2(a) = \tau_\mathcal{X}(b)\right\}$ with the projection $\pi_0^{2,*}(\tau_\mathcal{X})(a,b) = a$.

2. SYSTEM REPRESENTATION

In this contribution we will focus on pde-systems with a boundary control input, which are represented by system equations of the type

$$\begin{aligned} \dot{x}^\beta &= f^\beta\left(X^i, x^\alpha, x_i^\alpha, x_{ij}^\alpha\right), \ \beta = 1, \ldots, n, \\ 0 &= g^\nu\left(X_\partial^i, x^\alpha, x_i^\alpha, u^\kappa\right), \ \nu = 1, \ldots, b \le n. \end{aligned} \quad (1)$$

The system equations (1) correspond to a set of second-order pde's with appropiate bc's. It is worth stressing that the time t does not correspond to a coordinate in this setting, it remains in the role of an evolution parameter for the pde's. The underlying geometric structure of the system equations is given by the bundles $\pi : \mathcal{X} \to \mathcal{D}$ and $\rho : \mathcal{U} \to \partial\mathcal{D}$, where the latter one is equipped with the coordinates u^κ, $\kappa = 1, \ldots, m$ and X_∂. To include the bc's in the geometric picture we introduce the pullback bundle $\iota^*(J^1(\mathcal{X})) \to \partial\mathcal{D}$ [2] with the inclusion map $\iota : \partial\mathcal{D} \to \mathcal{D}$. Thereby, we build the fibred product bundle $\iota^*(J^1(\mathcal{X})) \times \mathcal{U} \to \partial\mathcal{D}$ as an appropriate underlying geometric structure of the bc's. The geometric picture of the system (1) is given by a generalized vector field $f = f^\beta\partial_\beta : J^2(\mathcal{X}) \to \pi_0^{2,*}(\mathcal{V}(\mathcal{X}))$, with $f^\beta \in C^\infty\left(J^2(\mathcal{X})\right)$ explicitly depending on first and second-order derivative coordinates. This generalized vector field defines a submanifold $\mathcal{S} \subset \pi_0^{2,*}(\mathcal{V}(\mathcal{X}))$. Moreover, the generalized system vector field f is subject to the bc's with $g^\nu \in C^\infty\left(\iota^*(J^1(\mathcal{X})) \times \mathcal{U}\right)$, which define a submanifold $\mathcal{S}_\partial \subset \iota^*(J^1(\mathcal{X})) \times \mathcal{U}$.

For the bc's, we assume that the condition $\mathrm{rank}\left[\partial_\kappa g^\nu\right] = m$ [3] holds. Thus, by the implicit function theorem we can locally rewrite the bc's in the form

$$\begin{aligned} u^\kappa &= g_1^\kappa\left(X_\partial^i, x^\alpha, x_i^\alpha\right), \ \kappa = 1, \ldots, m, \\ 0 &= g_2^\xi\left(X_\partial^i, x^\alpha, x_i^\alpha\right), \ \xi = 1, \ldots b - m. \end{aligned} \quad (2)$$

Remark 1. We suppose that the system equations (1) are well-posed in the sense of Hadamard, i.e. there are suitable Banach spaces for the solutions and the inputs, and the solutions depend continuously on their initial data and the inputs. Additionally, we suppose the well-posedness of all pde-systems which occur in the following.

A solution of (1) for an input $\eta(t) : \mathbb{R}^+ \to \Gamma(\partial\mathcal{D}, \mathcal{U})$ is given by a map $\Phi(t) : \mathbb{R}^+ \times \Gamma(\mathcal{D}, \mathcal{X}) \to \Gamma(\mathcal{D}, \mathcal{X})$ with the properties

$$\gamma(t) = \gamma_t = \Phi_t(\gamma_0), \ \gamma_{t_1+t_2} = \Phi_{t_2} \circ \Phi_{t_1}(\gamma_0),$$

and the initial condition $\gamma_0 = \gamma(0)$. Moreover, Φ_t satisfies the system equations

$$\begin{aligned} \partial_t \Phi_t^\beta(\gamma_0)(X) &= f^\beta \circ j^2(\Phi_t(\gamma_0))(X), \\ 0 &= g^\nu \circ \left(\eta_t, j^1(\Phi_t(\gamma_0))(X_\partial)\right). \end{aligned}$$

After this brief introduction, we start with the system decomposition.

3. LOCAL DECOMPOSITION

This section is devoted to the decomposition of the pde-systems (1). More precisely, we are trying to find a

[2] Here, the total space manifold is defined as $\left\{(a,b) \in \partial\mathcal{D} \times J^1(\mathcal{X}) : \iota(a) = \pi^1(b)\right\}$ with the projection $\iota^*(\pi^1)(a,b) = a$.

[3] $[\cdot]$ denotes the associated matrix representation, and ∂_κ is a shortcut for $\partial/\partial u^\kappa$ as already mentioned in the introduction.

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