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Periodic event-triggered sliding mode control ☆

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ABSTRACT

In this paper, we propose the periodic event-triggering based design of sliding mode control (SMC) for the linear time-invariant (LTI) systems. In this technique, the triggering instants are generated by a triggering mechanism which is evaluated periodically at those time instants when the state measurements are available. So, the continuous state measurement, as it is usually needed in the continuous event-triggering strategy, is no longer required in this proposed triggering strategy. The main advantages of this triggering mechanism are: (1) a *uniform* positive lower bound for the inter event time is guaranteed and (2) this technique is more economical and realistic than its continuous counterpart due to the relaxation of continuous measurements. In this work, we use SMC to design the periodic event-triggering condition where SMC is designed in such a way that it allows periodic evaluation of triggering rule while guaranteeing the robust performance of the system. Moreover, an upper bound of the sampling period for the periodic measurements is also obtained in this design. Finally, the simulation results are given to demonstrate the design methodology and performance of the system with the proposed event-triggering strategy.

1. Introduction

In modern control theory, the computer controlled systems (CCS) have become an important area of research due to its wide use in almost all fields of its application. Many design techniques have been proposed in the literature to study the stability of CCS (Åström & Wittenmark, 1997). Of all these, the classical sample and hold technique is more popular because of its simplicity in the design and the implementation. However, this strategy suffers from the increased computational burden, inefficient energy utilization, etc., particularly in network based control systems due to unwanted periodic evaluations. As a consequence, several other control implementation techniques have gained more interest in the past one decade, e.g., event-triggering (Tabuada, 2007), intermittent control (Li, Feng, & Liao, 2007), hands-off control (Nagahara, Quevedo, & Nešić, 2016), etc. In this paper, we consider the event-triggering strategy to analyse the stability of CCS. In the event-triggering based design, the control signal is updated when it is demanded subject to some satisfactory system performance. This is generally achieved by designing a triggering condition which decides when to update the control signal through online monitoring policy. For details, the readers may refer to Årźen (1999), Åström and Bernhardsson (2002), Girard (2015), Lunze and Lehmann (2009), Tabuada (2007), and references cited therein.

In almost all the works on event-triggering, the continuous state is measured in the design of event-triggered control. This introduces some additional cost in the implementation of this strategy due to the need of extra hardware circuit and the sophisticated sensors. To overcome these difficulties, some variants of event-triggering strategy have been proposed in the literature such as self-triggering, Anta and Tabuada (2010), Behera and Bandyopadhyay (2015) and Wang and Lemmon (2009), co-design approach, Behera, Bandyopadhyay, and Reger (2016), Kumari, Bandyopadhyay, Behera, and Reger (2016) and Meng and Chen (2014). Though these methods offer significant advantages, some degradations in the system performance occur compared to the classical event-triggering based implementation. For instance, the inter event time (time interval between any two consecutive triggering instants) is reduced in the self-triggered design, the larger sampling periods in co-design technique may result in the violation of the event condition before it is detected, and so on. Nevertheless, these techniques may be useful to implement the triggering condition in some particular applications. Recently, a new technique namely periodic event-triggered control is proposed in Heemels, Donkers, and Teel (2013) where the event condition is evaluated periodically. This technique seems to be more

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appealing since the measurements in practice are available only at periodic intervals due to the constraints on sensors. However, compared to other existing methods, in this case, the selection of sampling period plays an important role to guarantee the stability of the system. But, most of the papers assume that an appropriate time period is given *a priori* in the design (Heemels et al., 2013). This may again pose some question on how to select the sampling period for the periodic event-triggering based design of controller.

A very few papers have addressed the robust performance of the event-triggered control system. Recently, the sliding mode control (SMC) based event-triggering strategy is proposed to guarantee the robust performance of the system (Behera & Bandyopadhyay, 2014, 2016, 2017; Wen, Huang, Yu, Chen, & Zeng, 2017; Wu, Gao, Liu, & Li, 2017). It is shown that the similar (in the sense of *practical stability*) robust performance of the system, like in continuous implementation of SMC, can be achieved with the event-triggering strategy (Bartoszewicz, 1998; Draženović, 1969; Edwards & Spurgeon, 1998; Levant, 1993; Su, Drakunov, & Özgüner, 2000; Utkin, 1977). So, motivated by this, in this paper we use SMC to design the periodic event-triggering strategy for linear systems.

In this paper, we propose an SMC based event-triggering strategy using only the periodic measurements. The main advantage of the method is that the continuous evaluation of triggering rule is avoided, and also a (uniform) positive lower bound for the inter event time is guaranteed. Moreover, the periodic event-triggered SMC ensures the robust performance of the system with any desired steady-state bound. In this strategy, both the triggering rule and SMC are designed simultaneously such that the stability of the event-triggered system is ensured while satisfying event-triggering constraints. This is achieved due to the fact that SMC is always redesigned in event-triggering strategy. We also provide an upper bound on the sampling period for the design of periodic event-triggered SMC. In our design, it is shown that once the switching gain of SMC is designed, the sampling period can be chosen satisfying some design constraints.

In this paper, first the design of continuous event-triggering based SMC is proposed for the multi-input multi-output (MIMO) systems. A complete analysis for the stability of MIMO linear systems with event-triggered SMC is given. The sufficient condition for the stability of the closed loop system is obtained in this event-triggering strategy. Then, the periodic event-triggering mechanism is proposed motivated by the continuous triggering rule in which the triggering condition is evaluated periodically for some given time period. Then, using this triggering mechanism, we propose sufficient condition for periodic event-triggered SMC to ensure the stability of the closed loop system. Now, summarizing all these, the main contributions of the paper are as follows:

(i) First, the event-triggering based SMC is developed using continuous state measurement for MIMO systems. Here, we provide a sufficient condition and propose a triggering rule for robust stability of the closed-loop system. It is also shown that the proposed triggering rule generates a triggering sequence which is Zeno free.
(ii) Then, the periodic event-triggering strategy with SMC is developed in which the continuous evaluation of triggering rule is relaxed. Due to this, the error in the plant may increase beyond the threshold value set in the triggering condition in between two sampling instants. To ensure the stability of the system under this condition, the explicit bound of the error is obtained for some given sampling period.
(iii) The selection of sampling period is also given at which the triggering rule is evaluated periodically. It is shown that for every such sampling period the stability constraint of the system is met. Here, we provide an upper bound of the sampling period such that the closed loop system is stable for every sampling period less than or equal to this upper bound. This is achieved by designing SMC and triggering rule to meet the design constraint.

The rest of the paper is organized as follows. In Section 2, the preliminaries on the design of SMC are presented. The design of continuous event-triggered SMC is presented for an MIMO system in Section 3. Also, the motivation for periodic event-triggering strategy is discussed in this section. In Section 4, the design of periodic event-triggered SMC is proposed. In this section, we propose the periodic event-triggering mechanism for some sampling period. Then, we provide an upper bound of the sampling period to design for the periodic event-triggering strategy. The stability of the closed-loop system within some appropriate domain is also presented. Finally, simulation results are given in Section 5 which is followed by some concluding remarks in Section 6.

1.1. Notation

$\mathbb{R}$ and $\mathbb{Z}$ denote the set of all real numbers and integers, respectively. Similarly, $\mathbb{R}_{>0}$ and $\mathbb{R}_{\geq 0}$ (respectively, $\mathbb{Z}_{>0}$ and $\mathbb{Z}_{\geq 0}$) denote the set of all positive and nonnegative real numbers (integers), respectively. The maximum (minimum) eigenvalue of a matrix is denoted by $\lambda_{\max}\{\cdot\}$ ($\lambda_{\min}\{\cdot\}$). For any vector $z \in \mathbb{R}^n$, the 1-norm is denoted by $\|z\|_1$. Similarly, the Euclidean (2-) norm is represented as $\|z\|$ for the vector $z \in \mathbb{R}^n$. The induced 2-norm of a matrix $A \in \mathbb{R}^{m\times n}$ is defined as $\|A\| := \sup\{\|Ax\| : x \in \mathbb{R}^n \text{ with } \|x\| = 1\}$. This norm can also be represented by $\|A\| = \sqrt{\lambda_{\max}\{A^\top A\}}$.

2. Preliminaries

In this section, the preliminaries on the design of SMC for linear time-invariant (LTI) system are presented.

Consider a MIMO LTI system as

$$\dot{\tilde{x}} = \widetilde{A}\tilde{x} + \widetilde{B}(u + d), \qquad \tilde{x}(0) =: \tilde{x}_0 \in \mathbb{R}^n. \tag{1}$$

Here, $\tilde{x} \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ is the control input to the system. The external disturbance, $d(t)$, is assumed to be matched with respect to control input. The following assumptions are now stated which are valid throughout the paper.

Assumption 1. The pair $(\widetilde{A}, \widetilde{B})$ is controllable.

Assumption 2. The disturbance is bounded for all time with a known bound. So, there exists a $d_0 > 0$ such that $\sup_{t\geq 0}\|d(t)\| \leq d_0$.

At first, the system (1) is transformed into regular from[1] to design SMC. The system under the transformation $x = T_r\tilde{x}$ is given as

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 \tag{2a}$$
$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + u + d \tag{2b}$$

where $x_1 \in \mathbb{R}^{n-m}$ and $x_2 \in \mathbb{R}^m$. The system (2) can also be rewritten in compact form as

$$\dot{x} = Ax + B(u + d)$$

[1] Any minimal LTI system can always be transformed into regular form by a suitable nonsingular transformation. One such transformation is $T_r = \begin{bmatrix} I_{n-m} & -\widetilde{B}_1\widetilde{B}_2^{-1} \\ 0 & \widetilde{B}_2^{-1}\end{bmatrix}$. Here, $\widetilde{B}_1$ and $\widetilde{B}_2$ are obtained as $\widetilde{B} = \begin{bmatrix}\widetilde{B}_1 \\ \widetilde{B}_2\end{bmatrix}$ with $\widetilde{B}_2 \in \mathbb{R}^{m\times m}$ being invertible as rank $\widetilde{B} = m$.