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Representation and network synthesis for a class of mixed quantum–classical linear stochastic systems[☆]

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ABSTRACT

The purpose of this paper is to present a network realization theory for a class of mixed quantum–classical linear stochastic systems. Two forms, the standard form and the general form, of this class of linear mixed quantum–classical systems are proposed. Necessary and sufficient conditions for their physical realizability are derived. Based on these physical realizability conditions, a network synthesis theory for this class of linear mixed quantum–classical systems is developed, which clearly exhibits the quantum component, the classical component, and their interface. An example is used to illustrate the theory presented in this paper.

1. Introduction

Linear systems are of basic importance to classical control engineering, and also arise in the modeling and control of quantum systems; see, e.g., Dong and Petersen (2010), Gardiner and Zoller (2004), Gough and James (2009), Gough and Zhang (2015), Jacobs (2014), Mirrahimi and van Handel (2007), Nurdin and Yamamoto (2017), Sayed Hassen, Heurs, Huntington, Petersen, and James (2009), Wang and Dong (2017), Wang, Gao, and Dong (2017), Wilson et al. (2015), Wiseman and Milburn (2010), Zhang, Grivopoulos, Petersen, and Gough (2018), Zhang and James (2011), Zhang and James (2012) and Zhang, Liu, Wu, Jacobs, and Nori (2017). A classical linear system described by the state space representation can be realized using electrical components by linear electrical network synthesis theory, see Anderson and Vongpanitlerd (1973). Linear quantum optical systems may be described by linear quantum differential equations in the Heisenberg picture of quantum mechanics, Gardiner and Zoller (2004), Gough and James (2009), Gough and Zhang (2015), James, Nurdin, and Petersen (2008), Nurdin, James, and Petersen (2009), Nurdin and Yamamoto (2017), Wang, Nurdin, Zhang, and James (2013), Wilson et al. (2015), Wiseman and Milburn (2010) and Zhang et al. (2018). Such quantum linear systems described by the state space representation can be built by optical cavities, degenerate parametric amplifiers (DPA), phase shifters, beam splitters, and squeezers, etc.; interested readers may refer to Bachor and Ralph (2004), Leonhardt (2003), Nurdin, James, and Doherty (2009) and Nurdin and Yamamoto (2017) for a more detailed introduction to these optical devices. Quantum technologies often comprise quantum systems interconnected with classical (non-quantum) devices, which means that the two types of systems may be connected as an integral whole (called mixed quantum–classical systems in this paper) by appropriate interfaces that convert quantum signals to classical signals, and vice-versa. Traditionally, such quantum optical networks would be implemented on an optical table. However, it is now becoming possible to consider implementation in semiconductor chips, (Beausoleil, Keukes, Snider, Wang, &Williams, 2007; O'Brien, Furusawa, and Vuckovic, 2009; Wang et al., 2013).

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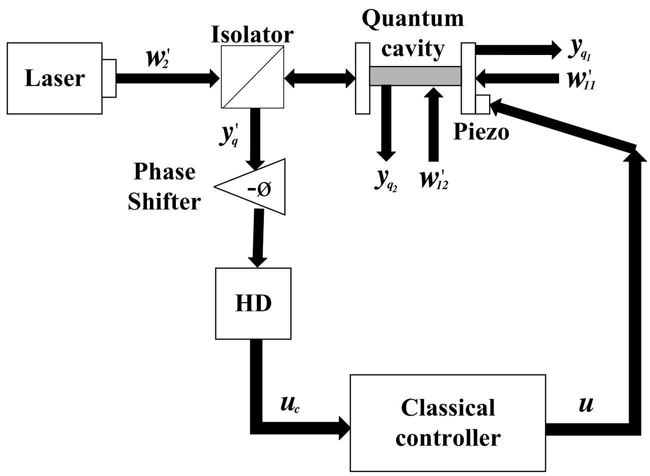

Fig. 1. A mixed quantum–classical system (Cavity locking feedback control loop) studied in Sayed Hassen et al. (2009).

In classical control engineering, many methods have been developed for designing controllers that meet various control specifications. The design process begins with some form of specification for the system, and concludes with a physical realization of the controller that meets the specifications. Often, mathematical models for the controller are used in the design process, such as state space equations for the controller. These state space equations may result from a mathematical optimization procedure, such as H^∞, LQG, or some other procedure. The process of going from such mathematical models to the desired physical systems is a process of *synthesis* or *physical realization*, part of the design methodologies widely used in classical engineering Anderson and Vongpanitlerd (1973). Analogous design issues are beginning to present themselves in quantum technology. A quantum control system often has both quantum and classical components. Indeed, in measurement-based feedback control, a classical controller is used to control a quantum plant. That is, a quantum control system is often a mixed quantum–classical system. Fig. 1 illustrates an example of a mixed quantum–classical linear system studied in Sayed Hassen et al. (2009). In this measurement-based feedback control system, a Fabry–Perot optical cavity Bachor and Ralph (2004), Nurdin et al. (2009) and Walls and Milburn (2008), which is described quantum-mechanically, is connected to a classical controller via a homodyne detector (HD) and a piezo-electric actuator Wiseman and Milburn (1993, 2010). The light field (quantum signal) reflected from the cavity is first separated from the incoming laser beam by an optical isolator, and then is detected by a HD (a quantum-to-classical converter), thus yielding a photocurrent which is a classical signal. The classical controller processes such classical signals to generate a classical control input u, which is then fed back to regulate the optical path length of the cavity via the piezo-electric actuator in order to actuate the resonant frequency of the cavity. Interested reader may refer to Sayed Hassen et al. (2009) for more details.

The purpose of this paper is to propose canonical representations for a class of linear stochastic differential equations that may describe mixed quantum–classical systems and then develop a network synthesis theory for such class of equations that reveals in a clear way the internal structure of a mixed quantum–classical system. Furthermore, arbitrary linear stochastic differential equations for mixed systems need not correspond to a physical system, and so we derive conditions ensuring that they do; that is, physical realizability. This work generalizes and extends earlier work James et al. (2008), Nurdin (2011), Wang, Nurdin, Zhang, and James (2012). In Wang et al. (2012), we only consider a *standard* model for mixed quantum–classical linear stochastic systems for the design process. However, in this paper, we will investigate a more general model for the physical realization of mixed quantum–classical linear stochastic systems.

The rest of the paper is organized as follows. Section 2 introduces some concepts about classical and quantum random variables as well as probabilities, briefly describes closed quantum harmonic oscillators, and also gives a brief overview of linear non-commutative stochastic systems and non-demolition conditions. Section 3 proposes two models of mixed quantum–classical linear stochastic systems for the design process and presents a connection between these models. Section 4 presents physical realizability definitions and constraints for the two models defined in Section 3, respectively. Section 5 develops a network synthesis theory for a mixed quantum–classical system. Section 6 presents a potential application of the main results of Section 5. Finally, Section 7 gives the conclusion of this paper.

2. Preliminaries

2.1. Notation

The notations used in this paper are as follows. The imaginary unit is $i = \sqrt{-1}$. The commutator of two operators A and B is defined by $[A, B] = AB - BA$. If x and y are column vectors of operators, the commutator is defined by $[x, y^T] = xy^T - (yx^T)^T$. If $X = [x_{jk}]$ is a matrix of linear operators or complex numbers, then $X^\# = [x_{jk}^*]$ denotes the operation of taking the adjoint of each element of X, and $X^\dagger = [x_{jk}^*]^T$. We also define $\Re(X) = (X + X^\#)/2$ and $\Im(X) = (X - X^\#)/2i$. The symbol I_k denotes the $k \times k$ identity matrix, $0_{j\times k}$ denotes the $j \times k$ zero matrix and $0_k \equiv 0_{k\times k}$. Let $J = [0\ 1; -1\ 0]$ and $\text{diag}_k(M)$ denote a block diagonal matrix with the square matrix M appearing k times on the diagonal block. A symplectic matrix V of dimension $2k\times 2k$ is a real matrix satisfying $V\Theta_k V^T = \Theta_k$, where $\Theta_k = \text{diag}_k(J)$. We set $\hbar = 1$ throughout this paper.

2.2. Classical and quantum random variables

A classical random variable, usually written as X, is a variable whose possible values are numerical outcomes of a random phenomenon. A random variable X with mean $\mu = \mathbf{E}[X]$ and variance $\sigma^2 = \mathbf{E}[(X-\mu)^2]$ is said to be *Gaussian* if its probability distribution function F is Gaussian, i.e.,

$$F(a < X \le b) = \int_a^b p_X(x)dx, \quad \forall -\infty < a < b < \infty, \tag{1}$$

where $p_X(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2})$ is of course the well-known Gaussian probability density function.

In quantum physics, a quantum random variable A is an operator defined on a Hilbert space $\mathfrak{H}$. In particular, if A is self-adjoint, it is called an *observable* and can be used to represent some physical quantity. Because an observable is self-adjoint, by the spectral theory, its spectra are real numbers. Actually, an observable can be physically measured to generate outcomes which are real numbers. On the other hand, a quantum *state* ψ encodes an experimenter's knowledge or information about some aspect of reality and is given mathematically as a vector of $\mathfrak{H}$, permitting the calculation of expected values of quantum random variables. If an observable A is measured on a quantum system prepared in the state ψ, then its mean value is given by the inner product $\langle\psi, A\psi\rangle = \int_{-\infty}^{\infty}\psi(q)^*A\psi(q)dq$. In quantum mechanics, the Dirac "ket" notation $|\psi\rangle$ is always used to denote a pure quantum state ψ. The adjoint of $|\psi\rangle$ is the "bra" vector $\langle\psi|$. Then, we can write