Automatica 96 (2018) 98–109

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Backstepping stabilization of a linearized ODE–PDE Rijke tube model[☆]

Gustavo Artur de Andrade[a,*], Rafael Vazquez[b], Daniel Juan Pagano[a]

[a] Department of Automation and Systems, Universidade Federal de Santa Catarina, 88040-900, Florianópolis, SC, Brazil
[b] Department of Aerospace Engineering, Universidad de Sevilla, Camino de los Descubrimientos, s.n., 41092, Sevilla, Spain

ARTICLE INFO

Article history:
Received 17 October 2017
Received in revised form 5 March 2018
Accepted 26 May 2018

Keywords:
Backstepping
Distributed systems
Partial differential equation
Rijke tube
Stabilization
Thermoacoustic instability

ABSTRACT

The problem of boundary stabilization of thermoacoustic oscillations in the Rijke tube is investigated using the backstepping method; as a first step, this work only considers the full-state design. This system consists of a vertical tube open at both ends and a heater placed in the lower half of the tube. To study this problem we consider that the mathematical model takes the form of 2×2 linear first-order hyperbolic partial differential equations (PDEs) with a point source term (induced by the Dirac delta distribution) on the right hand side, plus the coupling of an ordinary differential equation (ODE), and control input at one boundary condition. The presence of the Dirac delta distribution implies that the system solution has a discontinuity on a point of the domain, but is continuous everywhere else. We use a coordinate transformation to rewrite the equations into a system of four transport PDEs convecting in opposite directions and to translate the discontinuity to the boundary conditions. Then, a full state feedback backstepping controller is designed to exponentially stabilize the origin. However, the model is non-strict-feedback making unfeasible the use of standard backstepping designs. This issue is tackled by formulating a well-posed and invertible integral transformation with Volterra and Fredholm terms that maps the Rijke system into a target system with desirable stability properties. An exact piecewise-differentiable expression for the kernels of this transformation is found, allowing us in turn to derive an explicit feedback control law. Simulation results are presented to illustrate the effectiveness of the proposed control design.

1. Introduction

In this paper, we investigate the boundary stabilization of thermoacoustic oscillations in the Rijke tube. This system consists of a vertical tube, typically made of glass, open at both ends and with a heating element (usually a resistive Nichrome heater) placed towards the lower end. The air that traverses the heating zone expands, causing a sudden local pressure increase. The pressure propagates acoustically along the tube and returns, ultimately influencing itself at the heating area. Under the right conditions, the coupling between the acoustic and heat release dynamics becomes unstable. This instability manifests itself as a stable limit cycle resulting in audible pressure oscillations. A speaker placed a slight distance under the tube is used as an actuator to suppress the oscillations while a microphone near the top of the tube is typically used as sensor.

Thermoacoustic instability phenomena are often encountered in steam and gas turbines, industrial burners, and jet and ramjet engines (Krstic, Krupadanam, & Jacobson, 1999). In such applications, the thermoacoustic instabilities are also called combustion instabilities and are notorious difficult to model and study due to the additional complexity of combustion dynamics (Epperlein, Bamieh, & Astrom, 2015). The absence of combustion process in the Rijke tube makes the modeling and system analysis significantly more tractable, yet many of the feedback control issues appearing in combustion instabilities are also present in the Rijke tube. In this context, this experiment provides an accessible platform to explore and study thermoacoustic instabilities and their control.

The Rijke tube is often modeled by the Euler equations of gas dynamics, resulting in a 3×3 quasilinear first-order hyperbolic system, with the control variable appearing at the left boundary condition. Interestingly, the Dirac delta distribution appears on the right hand side of these equations, implying that the system solution could have a discontinuity on a point inside the domain, and be continuous everywhere else. The Dirac delta distribution is used to model the heat power added from the in-domain heating element into the gas. In turn, the heat release dynamics is given by a first-order ordinary differential equation (ODE) relating the dependence of released coil-to-gas heat power on the gas velocity. Depending on the steady-state conditions around which the model

☆ This work has been partially funded by the following projects: CNPq 150410/2017-4 and MTM2015-65608-P (financed by the Spanish Ministry of Economy and Competitiveness). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Thomas Meurer under the direction of Editor Miroslav Krstic.

* Corresponding author.
E-mail addresses: gustavo.artur@posgrad.ufsc.br (G.A. de Andrade), rvazquez1@us.es (R. Vazquez), daniel.pagano@ufsc.br (D.J. Pagano).

https://doi.org/10.1016/j.automatica.2018.06.034
0005-1098/

is linearized, one equation is decoupled from the others and therefore can be dropped from consideration. The resulting linearization is a linear first-order 2 × 2 partial differential equation (PDE) that behaves like a wave equation with the heat release fluctuation ODE acting as a point source term. The linearization of this model leads to the system considered throughout this paper.

Many authors contributed to the stabilization of thermoacoustic instabilities. The contributions range from phase shift controllers (Heckl, 1988) to linear–quadratic regulator controllers (Gelbert, Moeck, Paschereit, & King, 2012) or adaptive controllers (Banaszuk, Ariyur, & Krstic, 2004; Krstic et al., 1999; Riley, Park, Dowling, Evesque, & Annaswamy, 2004). Most of these works use a finite dimensional approximation of the system to design controllers. Recent approaches took into account distributed feature of the system, for example either by using a proportional–integral delayed control law (de Andrade, Vazquez, & Pagano, 2017; Zalluhoglu, Kammer, & Olgac, 2016), or by a Riemann invariant approach (de Andrade, Vazquez, & Pagano, 2016). The infinite dimensional backstepping method (Krstic & Smyshlyaev, 2008b) was also considered in the stabilization of thermoacoustic instabilities (see Bresch-Pietri & Krstic, 2014; Guo & Guo, 2013, and references therein). However, these authors assume several simplifications to provide a simpler model suitable for applying the standard backstepping method.

In this paper, we develop a full state feedback control law to stabilize thermoacoustic instabilities in the linearized Rijke tube model using the backstepping method. Different from the results of Bresch-Pietri and Krstic (2014) and Guo and Guo (2013), our approach takes into account a more realistic model, including the heat release dynamics and the downstream and upstream part of the tube, which requires the use of an integral transformation with Volterra and Fredholm terms. We start by introducing the characteristics coordinates, reformulating the plant in the form of transport PDEs convecting in opposite directions and coupled at the boundaries. Then, we break the spatial domain into two intervals: before and after the discontinuity point arising from the Dirac delta distribution – which is replaced by adequate boundary conditions – and consider different state variables for the states that represent the same physical quantity but are in different spatial intervals. By a change of coordinates we rescale these intervals so that all states evolve on the same domain. The resulting system, which is used to design the backstepping controller, is a 4 × 4 system of transport PDEs coupled at the boundaries, plus a linear ODE. The proposed backstepping control design achieves exponential stability of the zero equilibrium without the need to impose any conditions on the parameters of the system, unlike the results of de Andrade et al. (2017) and Zalluhoglu et al. (2016), which only guarantee input–output stability. The plant is mapped to a target system using a combination of Volterra and Fredholm transformations, instead of the usual Volterra-only mapping, because the plant is non-strict-feedback (see Chen, Vazquez, & Krstic, 2017 or Coron, Hu, & Olive, 2016 for other examples). The target system is designed by removing a part of the coupling terms at the boundary conditions, ensuring stability. Well-posedness of the kernel equations is proved by the method of characteristics, finding an explicit piecewise-differentiable solution; interestingly, the number of pieces of the solution and thus the complexity of the control law depends on the position of the heat release element. Moreover, we provide numerical simulations to show the effectiveness of the method for two different positions of this element.

The backstepping method has proved itself a very useful tool to stabilize PDE systems and in particular hyperbolic plants. Speaking of the latter, and starting with scalar hyperbolic equations (Krstic & Smyshlyaev, 2008a), successive extensions have been able to deal with 2 × 2 systems (Coron, Vazquez, Krstic, & Bastin, 2013) – with explicit designs in the constant-coefficient case, as shown in Vazquez and Krstic (2014) –, $n+1$ systems (Di Meglio, Vazquez, & Krstic, 2013) and finally general linear hyperbolic systems (two approaches exist, see Auriol & Di Meglio, 2016; Hu, Meglio, Vazquez, & Krstic, 2016, also Coron, Hu, & Olive, 2017). Other results include adaptive controllers, see for instance Anfinsen and Aamo (2017) and Yu, Vazquez, and Krstic (2017). These developments should not be seen as completely separate from backstepping designs for reaction–diffusion equations; recent works have established that there is a close relationship between parabolic and hyperbolic designs (see Vazquez & Krstic, 2017). Controllers for PDE–ODE systems have already been explored (see for instance Aamo, 2012; Hasan, Aamo, & Krstic, 2016; Tang & Xie, 2011), however previous results consider strict-feedback connections with the ODE subsystem, typically at the uncontrolled domain end.

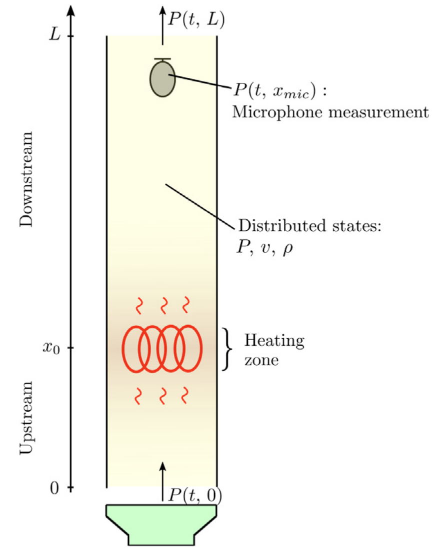

Fig. 1. Schematic of the Rijke tube with a heating element toward the bottom.

This paper is organized as follows. In Section 2, the Rijke tube model is described. Section 3 introduces the target system and the backstepping transformation. Numerical simulations are provided in Section 4. Concluding remarks and future directions are stated in Section 5.

2. The Rijke tube model

The phenomenon of thermoacoustic oscillations in the Rijke tube (see Fig. 1) can be captured using an one-dimensional model of compressible gas dynamics (de Andrade et al., 2017; Epperlein et al., 2015):

$$\partial_t \rho(t, x) + v(t, x)\partial_x \rho(t, x) + \rho(t, x)\partial_x v(t, x) = 0, \tag{1}$$

$$\partial_t v(t, x) + \partial_x v(t, x) + \frac{1}{\rho(t, x)}\partial_x P(t, x) = 0, \tag{2}$$

$$\begin{aligned}\partial_t P(t, x) &+ \gamma P(t, x)\partial_x v(t, x)\\ &+ v(t, x)\partial_x P(t, x) = \bar{\gamma}\frac{1}{A}\delta(x - x_0)Q(t),\end{aligned} \tag{3}$$

where $t \in [0, +\infty)$ is the time, $x \in [0, L]$ is the space, δ is the Dirac delta distribution, ρ is the gas density, P is the pressure, and v is the velocity. The tube cross-section is given by A, γ is the adiabatic ratio, $x_0 \in (0, L)$ is the location of the heat release source, Q is the heat power release, and $\bar{\gamma} = \gamma - 1$.

The heat power release is modeled by

$$\tau \dot{Q}(t) = -Q(t) + l_w(T_w - T_{gas})(\kappa + \kappa_v\sqrt{|v(t, x_0)|}), \tag{4}$$