Automatica 96 (2018) 166–177

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Robust Kalman filtering for two-dimensional systems with multiplicative noises and measurement degradations: The finite-horizon case ☆

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ARTICLE INFO

Article history:
Received 10 December 2016
Received in revised form 23 May 2018
Accepted 11 June 2018

Keywords:
Robust Kalman filtering
Two-dimensional systems
Shift-varying systems
Parameter uncertainties
Measurement degradation

ABSTRACT

In this paper, robust Kalman filtering problem is investigated for a class of two-dimensional (2-D) shift-varying uncertain systems with both additive and multiplicative noises over a finite horizon. The measurement outputs suffer from randomly occurring degradations obeying certain probabilistic distributions, and the norm-bounded parameter uncertainties enter into both the state and the output matrices. The main aim of this paper is to design a robust Kalman filter such that, in the presence of parameter uncertainties and degraded measurements, certain upper bound of the generalized estimation error variance is locally minimized in the trace sense at each shift step. Recursion of the generalized estimation error variances for the addressed 2-D system is first established via the introduction of a 2-D identity quadratic filter, based on which an upper bound of the generalized estimation error variance is obtained. Subsequently, such an upper bound is minimized in the trace sense by properly designing the filter parameters. The design scheme of the robust Kalman filter is presented in terms of two Riccati-like difference equations which can be recursively computed for programmed applications. Finally, a numerical example is provided to demonstrate effectiveness of the proposed filter design method.

1. Introduction

Since its invention in 1960 (Kalman, 1960), the legendary Kalman filter has found successful applications in a variety of areas that include, but are not limited to, estimation, control, signal processing, guidance, navigation, as well as time series analysis. The traditional Kalman filter requires accurate knowledge of the system parameters, the linear structure of the underlying system and the exact statistics of the Gaussian noises. Such a requirement tends to be stringent since systems to be tackled may involve unmodeled dynamics caused by, for example, parameter variations. As shown in Anderson and Moore (2012), even small modeling errors could seriously deteriorate the filter performance. To improve the robustness of the traditional Kalman filters, both robust Kalman filters and H_∞ filters have been a recurring theme of research interests in control and signal processing communities, and a number of significant results have been reported in the past two decades or so, see e.g. Anderson and Moore (2012), Jain (1975), Li, Wei, Han, and Liu (2016), Liu, Wei, Song, and Liu (2016), Moheimani, Savkin, and Petersen (1998), Petersen and McFarlane (1991), Petersen and Savkin (1999), Shaked and de Souza (1995), Theodor and Shaked (1996) and Xie and Soh (1994). Different from the H_∞ filter whose aim is to guarantee a prescribed bound on the L_2 gain from the exogenous disturbances to the estimation errors, the main idea of the robust Kalman filtering is to obtain an upper bound on the estimation error variance and then minimize such a bound through appropriately designing the filter parameters (Fu, de Souza, & Luo, 2001; Ma, Wang, Hu, Bo, & Guo, 2010; Shaked, Xie, & Soh, 2001; Yang, Wang, & Hung, 2002).

It should be pointed out that, to date, almost all robust filters have been designed exclusively for classical one-dimensional (1-D) systems whose states evolve in one single direction (Ahn, Shi, & Karimi, 2016; Caballero-Águila, Hermoso-Carazo, & Linares-Pérez, 2015; Chen, Lams, & Li, 2014; Chen, Yu, Zhang, & Wang, 2014; Geng, Liang, Feng, Xu, & Pan, 2016; Huang, Ho, & Lu, 2012; Karimi, Zapateiro, & Luo, 2010; Li, Jia, & Du, 2015; Ma & Sun,

☆ This work was supported in part by the National Key R&D Program of China under Grant 2018YFB1003800, the National Natural Science Foundation of China under Grant 61673110, the Jiangsu Provincial Key Laboratory of Networked Collective Intelligence under Grant BM2017002, the Royal Society of the UK and the Alexander von Humboldt Foundation of Germany. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Juan I. Yuz under the direction of Editor Torsten Söderström.

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https://doi.org/10.1016/j.automatica.2018.06.044
0005-1098/

2011; Niu, Ho, & Li, 2010; Shi, Su, & Li, 2016; Wang & Nguang, 2016). In practice, there is another kind of systems, referred to as two-dimensional (2-D) systems (Fornasini & Marchesini, 1976; Givone & Roesser, 1972), whose signals are transmitted along two independent directions. As discussed in Bose (2003), Kaczorek (1985), Liu (1998) and Roesser (1975), 2-D systems are capable of modeling practical systems whose dynamic evolutions are defined on multiple independent variables with examples including sheet-forming, image processing and water stream heating procedures. Comparing to 1-D systems, the 2-D counterparts exhibit more complicated dynamical behaviors because of two-directional evolutions, which pose more challenges on the corresponding analysis and synthesis issues. So far, considerable research interest has been focused on 2-D systems and numerous efforts have been made to develop some basic concepts and theories for 2-D systems, see e.g. Ahn, Shi, and Basin (2015, 2018), Liang, Wang, and Liu (2011), Luo, Wang, Liang, Wei, and Alsaadi (2017), Meng, Jia, Du, and Yuan (2009), Wang, Liang, Wang, and Liu (2018a, b).

For decades, the filtering problem of 2-D systems has been an attractive topic of research and many different filter design schemes have been proposed in the literature. For example, the H_∞ filtering problem has been considered in Du, Xie, and Soh (2000) for 2-D shift-invariant systems based on the linear matrix inequality method. The 2-D Kalman filtering problem has been investigated in Zou, Sheng, Zhong, and Xu (2004) where the system is assumed to be precisely known. When it comes to the 2-D uncertain systems, unfortunately, the corresponding robust Kalman filtering problem has not been adequately studied yet, which is due probably to the mathematical difficulties. One of the primary challenges is to establish a novel approach for designing the 2-D robust Kalman filter which, unlike the H_∞ theory (Geromel, de Oliveira, & Bernussou, 2002; Wei, Qiu, Karimi, & Wang, 2014), focuses on developing the tightest upper bound (in the sense of trace) on the filtering error variance at each step. Furthermore, the two-directional evolution leads to the difficulty in formulating the recursion of the error variances in the 2-D space. It is also a complicated issue to find the tightest upper bound of the error variances through appropriately designing the filter gains in the 2-D cases.

Notice that, in most available literature, the 2-D systems have been assumed to be shift-invariant for analysis simplicity. Such an assumption is, of course, restrictive as time-varying or shift-varying parameters exist in almost all real-world systems. In addition, the transient behaviors of the target plant are often of major concerns instead of the stability-oriented issues due to the specific engineering applications. From these perspectives, it is theoretically important and practically significant to study the robust Kalman filter design problem for 2-D systems with shift-varying parameters and parameter uncertainties.

On another research front, the measurement degradation has been well recognized as a frequently encountered phenomenon resulting typically from communication constraints (e.g. limited network bandwidth) in networked environments. The degraded measurements, which include missing measurements (also called packet losses or dropouts) as a special case, usually occur in a random way because of the random nature of the network load fluctuations, unexpected sensor aging, intermittent failures and accidentally changed working conditions (Ma & Sun, 2011; Sahebsara, Chen, & Shah, 2007; Yalcin, Collins, & Hebert, 2007). Such a randomly occurring phenomenon can be modeled by a stochastic variable satisfying a certain probabilistic distribution (Dong, Wang, Ho, & Gao, 2010; Liu, Wang, He, & Zhou, 2015, 2016) that is more general than the conventional Bernoulli one. Note that, in the 1-D case, much research attention has recently been paid to the degraded measurements and the corresponding robust Kalman filtering problems have stirred some initial research interest, see e.g. Wang, Yang, Ho, and Liu (2005) and Yang, Wang, Feng, and Liu (2009). Nevertheless, as far as the error variance is concerned, the Kalman filter design problem for 2-D systems with measurement degradation has not been investigated yet, let alone the case where the parameter uncertainties and multiplicative noises are both taken into account.

Summarizing the above discussions, we come to the conclusion that there is a gap in studying the robust Kalman filter design problem for 2-D shift-varying uncertain systems with measurement degradation, which has proven to be a practically important yet technically challenging issue. *Some identified challenges are outlined as follows: (1) the dynamics of a 2-D shift-varying system is particularly complicated to be analyzed, not to mention the simultaneous presence of the parameter uncertainties and measurement degradations; (2) it is quite challenging to derive an upper bound on the generalized estimation error variance for the 2-D systems, since the traditional approaches for 1-D uncertain systems are no longer applicable; and (3) it is non-trivial to design the 2-D robust Kalman filter whose generalized estimation error variance is upper bounded and subsequently minimized in the trace sense.* As such, the primary objective of this paper is to handle the identified challenges and shed light on the development of the 2-D robust Kalman filtering scheme.

In this paper, the general Fornasini–Marchesini (F-M) second model is considered to depict the system whose states are to be estimated. Parameter uncertainties are presented in the form of unknown but norm-bounded matrices. The phenomenon of measurement degradation is characterized by random variable satisfying certain given probabilistic distribution. The main contributions of this paper can be highlighted as follows: *(1) the addressed 2-D system is comprehensive to comprise shift-varying parameters, additive and multiplicative noises, parameter uncertainties and measurement degradations; (2) the problem to be tackled is new and our attempt represents one of the first few to investigate the robust recursive filtering problem for 2-D systems; (3) a novel approach is developed to guarantee the existence of an upper bound for the generalized estimation error variance; and (4) the robust Kalman filter is designed which ensures the local minimum trace of the acquired upper bound.*

The rest of this paper is organized as follows. Section 2 formulates the robust Kalman filtering problem for the considered 2-D system. Section 3 provides an upper bound for the generalized estimation error variance, and the determination of the locally minimal one is presented in Section 4. A simulation example is given to illustrate the effectiveness of the theoretical results in Section 5. Finally, the conclusion is drawn in Section 6.

Notations. I and 0 represent respectively the identity matrix and the zero matrix with appropriate dimensions. $\{D_i\}_{i=1}^m$ indicates the set $\{D_1, D_2, \ldots, D_m\}$, and for a nonnegative integer K, the notation $[0\ K]$ represents the set $\{0, 1, \ldots, K\}$ for simplicity. For a matrix $A \in \mathbb{R}^{n\times m}$, $A^{(kl)}$ infers the (k, l)th entry of A with $k \in [1\ n]$ and $l \in [1\ m]$. The notation $X > Y$ means that $X - Y$ is a real, symmetric and positive definite. The superscript 'T' is the transpose of vectors or matrices. $\mathbb{E}\{\alpha\}$ and $\mathrm{Var}\{\alpha\}$ are the mathematical expectation and the variance for certain stochastic variable α, respectively.

2. Problem formulation and preliminaries

Consider the following 2-D shift-varying uncertain system described by the F-M second state–space model over a finite horizon