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Stability analysis of networked linear control systems with direct-feedthrough terms☆

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We consider networked control systems (NCSs) composed of a linear plant and a linear controller interconnected by packet-based communication channels with communication constraints. We are interested in the setup where direct-feedthrough terms are present in the plant and/or in the controller, a case that is largely ignored in the literature due to its inherent complexity and counterintuitive results in the analysis despite its relevance for important classes of controllers including Proportional–Integral (PI) regulators. This setup calls for a novel stability analysis, for which we take a renewed look at the concept of uniformly globally exponentially stable (UGES) scheduling protocols that turned out to be instrumental in earlier approaches. We provide a generalization of the UGES property, called $(D_\mathbf{P}, D_\mathbf{C})$-UGES with $D_\mathbf{P}/D_\mathbf{C}$ being the direct-feedthrough matrices of the plant/controller, respectively, and we present generic conditions on these direct-feedthrough terms $D_\mathbf{P}/D_\mathbf{C}$ such that the classical UGES property of scheduling protocols implies $(D_\mathbf{P}, D_\mathbf{C})$-UGES. This allows us to derive conditions leading to a maximally allowable transmission interval (MATI) such that stability of the overall NCS is guaranteed. In addition, it is shown that it is possible to get more tailored results for the well-known sampled-data (SD), round-robin (RR), and try-once-discard (TOD) protocols leading to less conservative conditions on the direct-feedthrough terms than the generic ones. We also introduce new $(D_\mathbf{P}, D_\mathbf{C})$-UGES scheduling protocols, designed to handle the direct-feedthrough terms in a more effective way than existing protocols. Our results are illustrated using the example of a batch reactor.

1. Introduction

In many control applications, including manufacturing plants, vehicles, and aircraft, communication is needed for the exchange of information and control signals between spatially distributed system components, such as supervisory computers, controllers, sensors, and actuators. When sensor and actuator data is communicated over a shared (wired or wireless) packet-based communication network, the system is called a *networked control system* (NCS). Such NCSs have received considerable attention in recent years (Gao, Chen, & Lam, 2008; Hespanha, Naghshtabrizi, & Xu, 2007; Yang, 2006; Zhang, Branicky, & Phillips, 2001). This interest is motivated by the many advantages their flexible architectures offer, such as reduced weight, volume and installation costs, and better maintainability, when compared to conventional control systems in which sensor and actuation data is transmitted over dedicated point-to-point (wired) links, see, e.g., Raji (1994). Additionally, wireless communication is able to overcome the physical limitations of employing wired links, which is very appealing in, for instance, intelligent transportation, see, e.g., Öncü, van de Wouw, Heemels, and Nijmeijer (2012), and remote surgery, see, e.g., Meng et al. (2004). On the other hand, the usage of packet-based networked communication comes also with the inevitable network-induced imperfections, such as varying delays, dropouts, varying transmission intervals, and so on. Moreover, as the communication network is often shared by multiple sensors and actuators, there is

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a need for so-called scheduling protocols, which govern the access of the nodes to the network.

To deal with all these network-induced phenomena, novel design and analysis approaches are required. A popular design approach for NCSs herein is the so-called emulation method, see, e.g., Carnevale, Teel, and Nešić (2007), Heemels, Teel, van de Wouw, and Nešić (2010), Nešić and Teel (2004), Postoyan, van de Wouw, Nešić, and Heemels (2014) and Walsh, Ye, and Bushnell (2002). The idea is to first design a continuous-time controller for the continuous-time plant while ignoring the communication constraints. Then, the controller is implemented via the packet-based communication network with its scheduling protocol and it is shown that stability properties are preserved when information is transmitted frequently enough. By using the concept of uniformly globally exponentially stable (UGES) scheduling protocols introduced in Nešić and Teel (2004), conditions leading to the determination of so-called maximal allowable transmission intervals (MATIs) guaranteeing overall stability or $\mathcal{L}_p$-gain performance of the NCS have been derived, see Carnevale et al. (2007) and Nešić and Teel (2004). In addition to this general setup, many extensions can be found in Dolk, Borgers, and Heemels (2017), Heemels, Borgers, van de Wouw, Nešić, and Teel (2013), Heemels et al. (2010), Heijmans, Borgers, and Heemels (2017), and the references therein.

It is interesting to observe that none of the aforementioned results in Carnevale et al. (2007), Dolk et al. (2017), Heemels et al. (2013), Heemels et al. (2010), Heijmans, Borgers et al. (2017), Nešić and Teel (2004), Postoyan et al. (2014), and Walsh et al. (2002) considered the inclusion of so-called *direct-feedthrough terms*, i.e., terms that allow a direct connection between the control input and the plant output (and vice versa) and that are essential to model classical controllers commonly used in the industry such as Proportional–Integral(–Derivative) (PI(D)) regulators and state feedback controllers, when *both* actuator and sensor signals are transmitted over the communication network. However, as we will show in this paper, this is not surprising as these direct-feedthrough terms lead to nontrivial difficulties in terms of modeling and analysis. In particular, the presence of the direct-feedthrough terms modifies the model of the network-induced error at transmissions making it, in contrast to Carnevale et al. (2007), Dolk et al. (2017), Heemels et al. (2013), Heemels et al. (2010), Heijmans, Borgers et al. (2017), Nešić and Teel (2004), Postoyan et al. (2014), and Walsh et al. (2002), dependent on plant and controller parameters. This complicates the analysis significantly and leads to various counterintuitive results, as will be highlighted throughout the paper. As a result, a novel (stability) analysis is needed to address standard (UGES) scheduling protocols such as the sampled-data (SD) (which updates all network nodes simultaneously), round-robin (RR) (which assigns access to the network in a cyclic manner), and try-once-discard (TOD) (which gives access to the node with the largest error) protocols.

Given the importance of PI(D) control and other control/plant structures with direct-feedthrough terms, we already addressed this so-called *direct-feedthrough problem* in our preliminary works (Heijmans, Postoyan, Nešić, Noroozi, & Heemels, 2016; Noroozi, Postoyan, Nešić, Heijmans, & Heemels, 2016). In particular, it was shown in Noroozi et al. (2016) that, for the case where *only* the controller contained direct-feedthrough terms, stability of *nonlinear* NCSs could still be guaranteed when using standard scheduling protocols such as the SD and RR protocols. In Heijmans et al. (2016), *linear* NCSs with direct-feedthrough terms in *both* the plant and the controller were studied, which introduces additional difficulties. It was shown that for the standard SD, RR, and TOD protocols, under certain conditions, the direct-feedthrough terms can be incorporated in the NCS stability analysis of Carnevale et al. (2007), Nešić and Teel (2004) and Walsh et al. (2002).

In this paper, we build upon our preliminary work (Heijmans et al., 2016) and again consider linear NCSs with direct-feedthrough terms in *both* the plant and the controller. We provide a generalization of the UGES property for scheduling protocols, called $(D_\mathbf{P}, D_\mathbf{C})$-UGES, where $D_\mathbf{P}$ and $D_\mathbf{C}$ are the direct-feedthrough matrices of the plant and controller, respectively, and we present generic conditions on these direct-feedthrough terms $D_\mathbf{P}$ and $D_\mathbf{C}$ such that *any* UGES protocol in the classical sense is also $(D_\mathbf{P}, D_\mathbf{C})$-UGES, which is important to apply the stability analysis of Carnevale et al. (2007), Nešić and Teel (2004) and Walsh et al. (2002) to guarantee stability of the NCS. Moreover, it is shown that for the SD, RR, and TOD protocols these conditions can be made less conservative by exploiting the knowledge we have about the structure of the protocol. This will also lead to the counterintuitive result of the "smarter" TOD protocol not always being better than the RR protocol and that updating all the nodes simultaneously (exploiting the SD protocol) can be worse than updating the nodes one by one. Finally, we introduce new $(D_\mathbf{P}, D_\mathbf{C})$-UGES scheduling protocols, designed to handle the direct-feedthrough terms in a more effective way than the existing protocols in the case we do not have so-called mixed nodes, i.e., in the case that we only have nodes that are only related to the actuators and/or nodes that are only related to the sensors. These novel results significantly extend our preliminary work (Heijmans et al., 2016) as, in this paper, we provide full derivations and proofs, provide generic conditions for any UGES protocol to be $(D_\mathbf{P}, D_\mathbf{C})$-UGES and not only for the SD, RR, and TOD protocols as in Heijmans et al. (2016), introduce new ($(D_\mathbf{P}, D_\mathbf{C})$-UGES) scheduling protocols (called the SD+, the periodic switching, and maximal error switching scheduling protocol), and present various counterintuitive results, which, for instance, show that being "smarter" is not always the best solution. To illustrate our results we apply them to the benchmark example of a batch reactor.

The remainder of this paper is organized as follows. After presenting the necessary notation, the class of systems considered in this paper is described in Section 2 including a suitable hybrid model for the NCS. In Section 3, we briefly recapitulate the stability analysis of NCSs of Carnevale et al. (2007) and Nešić and Teel (2004), although slightly adapted to take into account the presence of the direct-feedthrough terms. In Section 4, we revise the concept of UGES scheduling protocols and provide generic conditions on the direct-feedthrough terms such that the stability analysis presented in Section 3 can be applied. These conditions are then improved for the well-known protocols SD, RR, and TOD in Section 5 and new scheduling protocols are introduced in Section 6. Finally, in Section 7, the batch reactor example illustrating our results is provided, and in Section 8 concluding remarks are given. All of the proofs are provided in Appendix A.

Notation: The set of real numbers is denoted by $\mathbb{R} := (-\infty, \infty)$ and the sets of non-negative real numbers and integers by $\mathbb{R}_{\geq 0} := [0, \infty)$ and $\mathbb{N} := 0, 1, 2, \ldots$, respectively. For vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$, we denote by $(v_1, v_2, \ldots, v_n)$ the vector $[v_1^\top \ v_2^\top \ \cdots \ v_n^\top]^\top$, and by $|\cdot|$ and $\langle \cdot, \cdot \rangle$ the Euclidean norm and the usual inner product, respectively. We use the notation $r^+ = r(t^+) = \lim_{\tau \downarrow t} r(\tau)$ where r is any left-continuous mapping from $\mathbb{R}$ to $\mathbb{R}^n$. The n by n identity and zero matrices are denoted by I_n and 0_n, respectively. When the dimensions are clear from the context, these notations are simplified to I and 0. For a symmetric matrix $A \in \mathbb{R}^{n\times n}$, $\lambda_{\min}(A)/\lambda_{\max}(A)$ denote the smallest/largest eigenvalue of A. A function $f : \mathbb{R}^m \to \mathbb{R}^n$ is said to be globally Lipschitz there exists $M > 0$ such that for all $x, y \in \mathbb{R}^m$ it holds that $|f(x) - f(y)| \leq M|x - y|$.

2. System description: The NCS model

In this section, the considered class of systems is introduced, where we in particular focus on the influence and impact of the direct-feedthrough terms on the NCS configuration as introduced in the literature, see also Heijmans et al. (2016).