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Opinion dynamics in social networks with stubborn agents: An issue-based perspective ☆

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ABSTRACT

Classic models on opinion dynamics usually focus on a group of agents forming their opinions interactively over a single issue. Yet generally agreement cannot be achieved over a single issue when agents are not completely open to interpersonal influence. In this paper, opinion formation in social networks with stubborn agents is considered over issue sequences. The social network with stubborn agents is described by the Friedkin–Johnsen (F–J) model where agents are stubborn to their initial opinions. Firstly, we propose a sufficient and necessary condition in terms of network topology for convergence of the F–J model over a single issue. Secondly, opinion formation of the F–J model is investigated over issue sequences. Our analysis establishes connections between the interpersonal influence network and the network describing the relationship of agents' initial opinions for successive issues. Taking advantage of these connections, we derive the sufficient and necessary condition for the F–J model to achieve opinion consensus and form clusters over issue sequences, respectively. Finally, we consider a more general scenario where each agent has bounded confidence in forming its initial opinion. By analyzing the evolution of agents' ultimate opinions for each issue over issue sequences, we prove that the connectivity of the state-dependent network is preserved in this setting. Then the conditions for agents to achieve opinion consensus over issue sequences are established. Simulation examples are provided to illustrate the effectiveness of our theoretical results.

1. Introduction

Recently, opinion dynamics has attracted much attention from various disciplines, such as applied mathematics, economics, social psychology, control theory, etc., due to its broad applications in modeling and explaining complex phenomena in social and artificial networks (Acemoglu, Ozdaglar, & ParandehGheibi, 2010; DeGroot, 1974; Frasca, Ishii, Ravazzi, & Tempo, 2015; Hegselmann & Krause, 2002; Lamport, Shostak, & Pease, 1982; Ravazzi, Frasca, Tempo, & Ishii, 2015). In a social network, agents form opinions on various political, economic or social issues according to the information they received from neighbors, which are determined by the network topology (Jadbabaie, Lin, & Morse, 2003; Li, Cornelius, Liu, Wang, & Barabási, 2017; Olfati-Saber & Murray, 2004; Ren & Beard, 2005) or the confidence/influence bound (Blondel, Hendrickx, & Tsitsiklis, 2009; Jing, Zheng, & Wang, 2017; Mirtabatabaei & Bullo, 2012). A fundamental question in opinion dynamics is: how do the network structure and opinions' initial distribution influence the diffusion and aggregation of scattered opinions in the process of opinion formation? In DeGroot (1974), a model is presented to characterize the process of a group of agents reaching opinion consensus on a common issue by pooling their subjective opinions, which is known as the DeGroot model. The interactions between agents are described by a stochastic matrix which can be regarded as the one-step transition probability matrix of a Markov chain, and some sufficient conditions for achieving opinion consensus are provided. To further investigate how the interpersonal influence contributes to the opinion formation, the work in Friedkin and Johnsen (1999) extends the DeGroot model by introducing a diagonal matrix which represents agents' susceptibilities to interpersonal influence. In the DeGroot model and the Friedkin–Johnsen (F–J) model, interactions between agents are specified by given networks. Different from these linear models, in Hegselmann and Krause (2002), the authors present a nonlinear model in which agents have bounded confidence on others. In the Hegselmann–Krause (H–K) model, two agents are said to be connected if and only if the difference between their opinions is smaller than a given

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confidence bound, which means that the network topology of the H–K model is state-dependent.

As the research in multi-agent systems continues, available tools to deal with opinion dynamics have been enriched considerably. The DeGroot model is further investigated both on continuous-time dynamics and switched topologies (Wang, Shi, Chu, Zhang & Zhang, 2004; Guan & Wang, 2017; Jadbabaie et al., 2003; Lin, Zheng, & Wang, 2017; Olfati-Saber & Murray, 2004; Ren & Beard, 2005). In addition, many complex scenarios are also considered, including asynchronous consensus (Xiao & Wang, 2008), time delays (Wang & Xiao, 2007), finite-time consensus (Wang & Xiao, 2010), leader-following framework (Wang, Jiang, Xie & Ji, 2009; Liu, Xie, & Wang, 2012; Ma, Zheng, Wu, & Wang, 2016), heterogeneous model (Zheng, Ma, & Wang, 2018; Zheng & Wang, 2014) and antagonistic interactions (Altafini, 2013), to name but a few. Based on the gossip algorithm (Boyd, Ghosh, Prabhakar, & Shah, 2006), randomized opinion dynamics is investigated in Acemoglu et al. (2010), Frasca et al. (2015), Li, Scaglione, Swami, and Zhao (2013) and Ravazzi et al. (2015), respectively. In Ghaderi and Srikant (2014), the equilibrium and convergence rate of the F–J model are investigated. Under the assumption that the adjacent matrix of the interpersonal influence network is an irreducible substochastic matrix with the row sum of at least one row strictly smaller than one, the authors transform the F–J model to a random walk, and the form of equilibrium is proposed based on the first hitting probabilities of the random walk. In Bindel, Kleinberg, and Oren (2011), the F–J model is interpreted as a best-response game, and the ratio between the optimal solution and the Nash equilibrium solution, which is defined as the price of anarchy, is discussed under both undirected and directed networks.

Most of the available literature on the F–J model focuses on opinion evolving over a single issue. Yet the F–J model does not in general converge to consensus over a single issue due to the presence of stubborn agents. Actually, the empirical evidence shows that consensus may be reached over a sequence of issues (Joshi, Dencker, Franz, & Martocchio, 2010). In practice, associations of individuals (such as small groups within firms, deliberative bodies of government, etc.) are usually constituted to deal with issues within particular issue domains which consist of deeply interdependent issues, especially repeatedly arising issues. In this scenario, individuals' opinions for interdependent issues are always correlated. Thus, extending the existing theories of opinion dynamics to issue sequences is necessary, and may uncover the underlying mechanism of opinion formation in the real world. Jia, Mirtabatabaei, Friedkin, and Bullo (2015) and Mirtabatabaei, Jia, Friedkin, and Bullo (2014) modify the F–J model and the DeGroot model to investigate the evolution of agents' self-appraisals over an issue sequence, respectively. In Parsegov, Proskurnikov, Tempo, and Friedkin (2017), the authors present a multidimensional extension of the F–J model in which agents' opinions for several interdependent issues evolve over time sequences.

Different from the works which focus on agents' self-appraisal dynamics (Jia et al., 2015; Mirtabatabaei et al., 2014) over sequences of independent issues, or evolution of agents' opinions for multiple interdependent but unordered issues over time sequences (Parsegov et al., 2017), we consider the opinion formation of the F–J model which evolves over both interdependent issue sequences and time sequences in this paper. Firstly, convergence of the F–J model is studied over a single issue. The existing results on convergence of the F–J model over a single issue usually require that the interpersonal influence network is undirected connected, or its adjacent matrix is strictly row-substochastic (Ghaderi & Srikant, 2014; Mirtabatabaei et al., 2014). We propose a milder sufficient and necessary condition in terms of network topology to guarantee that agents' opinions converge. Several properties of agents' ultimate opinions on a single issue are also provided. Secondly, we study opinion formation of the F–J model over a sequence of interdependent issues inspired by the path-dependence theory (Egidi & Narduzzo, 1997; MacKay, Masrani, & McKiernan, 2006; North, 1990; Page, 2006). By virtue of the inherent coherence between basic assumption of the F–J model and the path-dependence theory, we assume that the factor of each agent's cognitive inertia over two interdependent issues is equal to its stubborn factor. Then, each agent will form its initial opinion for the next issue by making a tradeoff between its initial opinion for the last interdependent issue and other agents' initial opinions for the next issue. The connections between the interpersonal influence network and the network which characterizes the relationship of agents' initial opinions for successive issues are established. Then the sufficient and necessary condition for the F–J model to achieve opinion consensus or form clusters over issue sequences is proposed. Finally, we consider the more general case in which an information assimilation mechanism is employed to weaken agents' interpersonal influence. We assume that each agent maintains a confidence bound in forming its initial opinions. This assumption is consistent with the reported echo-chamber effect (Dandekar, Goel, & Lee, 2013; Sunstein, 2012), i.e., people usually assimilate information in a selective way: they tend to give considerable weight to the information supporting their initial opinions, and dismiss the undermining information meanwhile. Connectivity preservation of the modified F–J model is analyzed. Then we derive the conditions for achieving opinion consensus.

The main difficulties for our analysis are twofold. On one hand, the evolution of agents' initial opinions over issue sequences is not directly determined by the interpersonal influence network and agents' stubborn extent, which increases complexity of our analysis from the graphical perspective. On the other hand, due to the fact that the evolution of agents' initial opinions over issue sequences is influenced by the evolution of their opinions over time sequences, connectivity preservation of the modified F–J model is more complex compared with connectivity preservation of the H–K model. As has been widely reported (Frasca et al., 2015; Friedkin, 2015; Ghaderi & Srikant, 2014), a social network with stubborn agents does not generally achieve consensus over a single issue. Our investigation shows that opinions of a group of individuals can achieve consensus over each path-dependent issue sequence,[1] even if agents are stubborn to their initial opinions. Simply put, repeatedly arising or interdependent issues lead to consensus. Our study also provides a theoretical explanation for the cohort effect (Joshi et al., 2010). Moreover, the evolution of opinions over issue sequences enhances the connectivity of the social network. As long as one of the partially stubborn agent's opinions can scatter to all the rest partially stubborn agents through partially stubborn or non-stubborn agents, opinion consensus can be reached, and all the partially stubborn agents form a star subgraph in the network which characterizes the relationship of agents' initial opinions for successive issues. The differences between opinion dynamics over issue sequences and a single issue give us a deeper understanding of opinion formation in social networks.

The rest of this paper is organized as follows. In Section 2, we introduce some notions on graph theory and the F–J model. In Section 3, convergence of the F–J model is studied over a single issue, and some properties of ultimate opinions are proposed. In Section 4, opinion formation of the F–J model is investigated over issue sequences. In Section 5, we consider opinion formation of the F–J model over issue sequences with bounded confidence. In Section 6, numerical simulations are worked out to illustrate the

[1] In the sequel, we say that an issue sequence is path-dependent or has the property of path-dependence if individual's opinions have the path-dependent property over it. A detailed explanation of path dependence will be presented below.