Automatica 96 (2018) 259–269

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Finite-time output feedback stabilization of lower-triangular nonlinear systems ☆

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ARTICLE INFO

Article history:
Received 23 June 2017
Received in revised form 18 April 2018
Accepted 15 May 2018

Keywords:
Finite-time stability
Finite-time observer
Output feedback stabilization
Nonlinear systems

ABSTRACT

The finite-time stabilization of a general class of lower-triangular nonlinear systems using output feedback is addressed in this paper. The system nonlinearities are only required to be Hölder continuous with output-dependent Hölder coefficients. A new constructive output-feedback, finite-time controller design is proposed based on a finite-time observer. Rigorous finite-time stability analysis is given for the closed-loop observer–controller system. A practical example of controlling a robotic model is adopted to illustrate the proposed method.

1. Introduction

Stabilization of nonlinear systems is of paramount importance in control theory. There are a good number of papers on this topic, such as local, semi-global, and global stabilization. See, for instance (Freidovich & Khalil, 2008; Isidori, 2000; Krstic, Kanellakopoulos, & Kokotovic, 1995; Praly & Jiang, 2004; Rios, Mera, Efimov, & Polyakov, 2017; Teel & Praly, 1995) and numerous references therein. Recently, some works are dedicated to the continuous finite-time stabilization of double integrators (Bernuau, Perruquetti, Efimov, & Moulay, 2015; Bhat & Bernstein, 1998; Hong, Huang, & Xu, 2001), chain of integrators (Lopez-Ramirez, Efimov, Polyakov, & Perruquetti, 2016), chain of power-integrators (Hong, 2002), lower-triangular systems (Hong, Jiang, & Feng, 2010; Li, Qian, & Ding, 2010; Li, Sun, Yang, & Yu, 2015), stochastic systems (Gao & Wu, 2016; Huang & Xiang, 2016a, b; Li, Zhao, & Liu, 2016), and the optimal finite-time stabilization (Haddad & L'Afflitto, 2016). The finite-time stabilization means that, under a suitably designed feedback controller, the state of a control system will identically equal zero after finite time, as opposed to asymptotic stabilization requiring to drive the state to zero asymptotically as time goes to infinity.

Although significant progresses have been made on the finite-time stabilization, except for Refs. Bernuau et al. (2015), Hong et al. (2001) and Lopez-Ramirez et al. (2016) that studied only for the chains of integrators and Ref. Li et al. (2010) for the nonlinear systems with restrictive growth conditions, a common feature of the existing works (Gao & Wu, 2016; Hong, 2002; Hong et al., 2010; Li et al., 2015, 2016) is that the finite-time stabilization results for general strongly nonlinear systems are obtained by using full-state information. Often, in practical applications, only partial-state information is available for the feedback controller design. This thus challenges us to develop new tools for the design of output-feedback controllers to solve the finite-time stabilization of general strongly nonlinear systems. In this paper, we take up this challenge by constructing an observer-based, output-feedback, finite-time controller. The considered class of nonlinear systems is transformable into the following lower-triangular form:

$$\begin{cases} \dot{x}_1(t) = x_2(t) + \phi_1(y(t)), \\ \dot{x}_2(t) = x_3(t) + \phi_2(y(t), x_2(t)), \\ \quad \cdots \\ \dot{x}_{n-1}(t) = x_n(t) + \phi_{n-1}(y(t), \ldots, x_{n-1}(t)), \\ \dot{x}_n(t) = \phi_n(y(t), \ldots, x_n(t)) + u(t), \end{cases} \tag{1.1}$$

where $x = (x_1, \ldots, x_n)^\top \in \mathbb{R}^n$ is the state, $\phi_i \in C(\mathbb{R}^i, \mathbb{R})$, $y = x_1$ is the output (measurement), u is the control (input). The observation and stabilization problems for the lower-triangular systems have been extensively investigated recently, see for example Andrieu, Praly, and Astolfi (2009), Hong et al. (2010), Krstic et al. (1995), Perruquetti, Floquet, and Moulay (2008), Praly and Jiang (2004) and

☆ This work is supported partly by the National Natural Science Foundation of China (No. 61403242), the Natural Science Research Program of Shaanxi Province-Key Program (No. 2016JZ023), the Fundamental Research Funds for the Central Universities (No. GK201702013), and the National Science Foundation (No. ECCS-1501044). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Hyungbo Shim under the direction of Editor Daniel Liberzon.

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https://doi.org/10.1016/j.automatica.2018.07.003
0005-1098/

Shen and Xia (2008), to list just a few. To the best of our knowledge, there is no paper addressing the finite-time stabilization problem for lower-triangular, high-order, strongly nonlinear systems by output feedback.

In this paper, to achieve the finite-time stabilization by means of output feedback, a finite-time observer is adopted for the output-feedback controller design. Finite-time observers for lower-triangular open-loop systems can be found in Du, He, and Cheng (2014), Duo, Qian, Yang, and Li (2013), Perruquetti et al. (2008), Rios et al. (2017), Sepulveda Galvez, Badillo-Corona, and Chairez (2015) and Shen and Xia (2008). However, the output feedback finite-time stabilization problem of the lower-triangular strongly nonlinear systems remains open. In this paper, we will give a solution to the output-feedback finite-time stabilization for the nonlinear lower-triangular system (1.1) under mild conditions.

In this paper, as in Ref. Andrieu et al. (2009), the nonlinear functions $\phi_i(\cdot)$'s in (1.1) are assumed to satisfy the following Hölder condition with output-dependent Hölder coefficients:

Assumption 1. There exists a function $\mathcal{L} \in C(\mathbb{R}, [0, \infty))$ such that

$$|\phi_i(y, x_2, \dots, x_i) - \phi_i(y, \hat{x}_2, \dots, \hat{x}_i)| \\ \leq \mathcal{L}(y)\left(|x_2 - \hat{x}_2|^{\alpha_{i2}} + \cdots + |x_i - \hat{x}_i|^{\alpha_{ii}}\right). \tag{1.2}$$

and

$$|\phi_i(y, 0, \dots, 0)| \leq \mathcal{L}(y)|y|^{\alpha_{i1}}, \quad i = 1, 2, \dots, n, \tag{1.3}$$

where $\alpha_{ij} \in (0, 1], j = 1, \dots, i, i = 1, \dots, n$.

In Ref. Praly and Jiang (2004), the output-feedback asymptotic stability is obtained by assuming the nonlinear functions $\phi_i(\cdot)$'s in system (1.1) are Lipschitz continuous with the output-dependent Lipschitz coefficients. In existing finite-time observer designs for open-loop systems (Perruquetti et al., 2008; Shen & Xia, 2008), the nonlinear functions $\phi_i(\cdot)$'s are assumed to be Lipschitz continuous with constant Lipschitz coefficients. Notice that Assumption 1 holds with all α_{ij}'s equal 1, when ϕ_i's are Lipschitz continuous and $\phi_i(0) = 0$.

As compared with Ref. Andrieu et al. (2009) where the (asymptotic) observer design problem for a class of open-loop systems is addressed, in this paper, we will study for the first time the output-feedback finite-time stabilization problem for a class of strongly nonlinear systems. The major difficulties encountered in this paper are the high-order stabilizer design for the strongly nonlinear lower-triangular systems, and the finite-time stability analysis of the observer-based closed-loop control systems. It does not appear easy to generalize the recursive state-feedback design method in Hong et al. (2010) to the out-feedback finite-time stabilization setting. A novel design based on the saturation strategy is developed to overcome these difficulties.

The remaining contents are organized as follows: In Section 2, we will first give a finite-time observer for system (1.1). A finite-time controller is then designed for system (1.1) by using saturation functions. The main result on finite-time stabilization is presented in this section. Section 3 presents the proof of our main result. A practical application to a robot is presented in Section 4 for the illustration. Some concluding remarks are given in Section 5.

2. Finite-time observer and output-feedback controller

In this section, we first design a finite-time observer for system (1.1). Then we present an observer-based, output-feedback, finite-time stabilizer design scheme for the system (1.1) by adopting the saturation method.

Inspired by Hong et al. (2001), Perruquetti et al. (2008) and Shen and Xia (2008), a finite-time observer for system (1.1) can be designed as:

$$\begin{cases} \dot{\hat{x}}_1(t;r) = \hat{x}_2(t;r) + \dfrac{k_1\left[r^n(y(t) - \hat{x}_1(t;r))\right]^{\theta_1}}{r^{n-1}} \\ \qquad +\phi_1(y(t)), \\ \dot{\hat{x}}_2(t;r) = \hat{x}_3(t;r) + \dfrac{k_2\left[r^n(y(t) - \hat{x}_1(t;r))\right]^{\theta_2}}{r^{n-2}} \\ \qquad +\phi_2(y(t), \hat{x}_2(t;r)), \\ \quad \vdots \\ \dot{\hat{x}}_n(t;r) = k_n\left[r^n(y(t) - \hat{x}_1(t;r))\right]^{\theta_n} \\ \qquad +\phi_n(y(t), \hat{x}_2(t;r), \dots, \hat{x}_n(t;r)) + u(t), \end{cases} \tag{2.1}$$

where $r > 0$, the function $[\cdot]^{\theta_i}$ is defined as $[\tau]^{\theta_i} = |\tau|^{\theta_i}\mathrm{sign}(\tau)$, $\theta_i = i\theta - (i-1), i = 1, 2, \dots, n, \theta \in (1 - 1/n, 1)$, and the constants k_i's are chosen such that the following matrix is Hurwitz:

$$K_{n\times n} = \begin{pmatrix} -k_1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_{n-1} & 0 & 0 & \cdots & 1 \\ -k_n & 0 & 0 & \cdots & 0 \end{pmatrix}_{n\times n}. \tag{2.2}$$

The parameters k_i's can be determined by pole placement technique. The similar finite-time observer can be found in Du et al. (2014) and Duo et al. (2013).

The terminal-sliding mode observer (Tan, Yu, & Ma, 2010) is also a finite-time observer. Compared with the finite-time observer proposed in this paper, the terminal-sliding mode observer is only valid for a special case of system (1.1) when $\phi_i = 0$ for each $i = 1, \dots, n-1$. Furthermore, the design functions in the finite-time observer of this paper is continuous, as opposed to discontinuous terminal-sliding mode observer.

To address the general observer-based, output-feedback, finite-time stabilization problem for the system (1.1) under general Assumption 1, we construct the following finite-time stabilizing controller (2.3) by adopting saturation functions:

$$u(t) = \rho\sum_{i=1}^{n} a_i\left[\mathrm{sat}_{M_i}\left(\rho^{n-i}\hat{x}_i(t;r)\right)\right]^{\beta_i}, \tag{2.3}$$

where $\rho > 0$ is a positive constant,

$$\beta_i = \frac{n\beta - (n-1)}{(i-1)\beta - (i-2)}, \quad i = 1, 2, \dots, n, \tag{2.4}$$

$\beta \in (1 - 1/n, 1)$, and a_i's are chosen such that the matrix below is Hurwitz:

$$A_{n\times n} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_n \end{pmatrix}_{n\times n}. \tag{2.5}$$

The saturation function $\mathrm{sat}_{M_i} : \mathbb{R} \to \mathbb{R}$ is defined by

$$\mathrm{sat}_{M_i}(\tau) = \begin{cases} -M_i, & \tau \in (-\infty, -M_i], \\ \tau, & \tau \in (-M_i, M_i), \\ M_i, & \tau \in [M_i, \infty), \end{cases} \tag{2.6}$$

where $M_i (1 \leq i \leq n)$ are positive constants to be specified in (3.45), that depend on the upper bounds of initial values of system (1.1).

Theorem 2.1. *Assume the matrices defined in (2.2) and (2.5) are Hurwitz, the nonlinear functions $\phi_i(\cdot)$'s in the system (1.1) satisfy Assumption 1. Then there exist $\tilde{\alpha}^* \in (0, 1)$, $\beta^* \in (0, 1)$, $\theta^* \in (0, 1)$, $r^* > 0$, and $\rho^* > 0$ such that for any $\alpha_{ij} \in [\tilde{\alpha}^*, 1]$, $\beta \in (\beta^*, 1)$, $\theta \in (\theta^*, 1)$, $r > r^*$, and $\rho > \rho^*$, the solutions of the system (1.1) and observer (2.1) satisfy*

$$\hat{x}_i(t;r) = x_i(t), \quad x_i(t) = 0, \quad \forall\, t > T, \ i = 1, 2, \dots, n, \tag{2.7}$$