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A fundamental control performance limit for a class of positive nonlinear systems☆

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ABSTRACT

A fundamental performance limit is derived for a class of positive nonlinear systems. The performance limit describes the achievable output response in the presence of a positive disturbance and subject to a sign constraint on the allowable input. An explicit optimal input is derived which minimises the maximum output response whilst ensuring that the minimum output response does not fall below a pre-specified lower bound. The result provides a fundamental performance standard against which all control policies, including closed loop schemes, can be compared. Implications of the result are examined in the context of blood glucose regulation for Type 1 Diabetes.

1. Introduction

Positive dynamical systems are frequently used to describe physical systems including those found in economics, chemical processes and biology (Farina & Rinaldi, 2011; Kaczorek, 2002). There exists a substantial literature on the modelling and properties of such systems (Berman, Neumann, & Stern, 1989; Farina & Rinaldi, 2011; Kaczorek, 2002; Krasnosel'skij, Lifshits, & Sobolev, 1989).

The current paper develops a fundamental performance limit for a class of positive nonlinear systems. The result is in the spirit of other existing results on performance limits which, inter-alia, give bounds on achievable performance subject to constraints on the model, control architecture and/or allowable signals (Chen, 1995; Freudenberg & Looze, 1985; Goodwin, Graebe, & Salgado, 2001; Horowitz, 1963; Middleton, 1991; Seron, Braslavsky, & Goodwin, 1997).

The result applies to a class of positive nonlinear systems in which the input is constrained to be positive whilst the output is required to satisfy a specified lower bound. Subject to these constraints, the goal is to minimise the maximum excursion of the output variable in response to a particular disturbance. A closed form solution to this optimisation problem is developed. The result provides a performance limit against which all other control policies can be compared.

An important case where the result is of relevance is in blood glucose regulation for type 1 diabetes management (Atkinson, Eisenbarth, & Michels, 2014; Chee & Fernando, 2007). A common model used in this area is a special case of the model class considered here. For this particular problem, the input signal is externally delivered insulin flow, which is necessarily positive; the disturbance corresponds to ingested food; the safety lower bound on the output arises from the need to avoid hypoglycaemia and the performance goal is to reduce long term hyperglycaemia (Aronoff, Berkowitz, Shreiner, & Want, 2004). The results presented here show that an impulse (bolus) of insulin is better than any other insulin injection policy applied at the same time or after. This also implies that it is better to apply an insulin bolus as soon as practical either prior to the meal or with the meal. This conclusion reinforces the common advice given by clinicians. Note that, in practice, a related dual wave administration of insulin is sometimes preferred due to robustness reasons — see Remark 29 later in the paper.

The current paper extends and embellishes several recent results aimed more directly at the diabetes problem (Goodwin, Medioli, Carrasco, King, & Fu, 2015; Phan et al., 2016; Townsend & Seron, 2018; Townsend, Seron, & Goodwin, 2017). The result presented here goes beyond the earlier results in several aspects. Firstly, the earlier results were obtained for a particular blood glucose model, whereas the results herein hold for a more general class of nonlinear systems. Secondly, earlier work depended on an assumption regarding the impulse response of a third order stable all-real-pole linear subsystem, i.e., the system connecting

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insulin flow to insulin effectiveness (in plasma). In this paper, the latter assumption is proven to hold for arbitrary stable all-real-pole linear systems. Thirdly, in previous work, the times when the maximum and minimum response occur were implicitly defined. In the current paper, the times are made explicit in terms of a prototype input signal. Fourthly, in Townsend and Seron (2018) and Townsend et al. (2017) the analysis was restricted to square wave (pulse) inputs. Here, arbitrary positive inputs are considered. In view of the above, both the result and the proofs in the current paper are significantly more general than in earlier work.

Delays are common in biological processes, meaning there is typically an unavoidable period between the time inputs are applied and when they become effective. Because of their importance, there exists a substantial literature on the impact of delays on performance and stability of control systems — see for example Gao and Olgac (2016, 2017), Gao, Kammer, Zalluhoglu, and Olgac (2015a, b) and Gao, Zalluhoglu, and Olgac (2014). The aforementioned results represent an important restriction on the achievable closed loop bandwidth for controllers such as those presented in Atlas, Nimri, Miller, Grunberg, and Phillip (2010), Bequette (2005, 2012), Cefalu and Tamborlane (2014), Chee and Fernando (2007), Doyle III, Huyett, Lee, Zisser, and Dassau (2014), El-Khatib, Russell, Nathan, Sutherlin, and Damiano (2010), Gondhalekar, Dassau, and Doyle III (2016), Harvey et al. (2010), Klonoff, Cobelli, Kovatchev, and Zisser (2009), Kovatchev et al. (2010), Kovatchev, Tamborlane, Cefalu, and Cobelli (2016), Kumareswaran (2012), Lee, Buckingham, Wilson, and Bequette (2009), Weinzimer et al. (2008) and elsewhere for the Type 1 diabetes problem. The key result of the current paper has a feedforward character, and thus closed loop stability issues do not arise. Nonetheless, we have explicitly accounted for time delays in the problem formulation and solution. Two potential sources of delay are considered, namely an unavoidable plant (or biological) delay and a control law delay, specifying the time when the control action departs from a chosen steady-state value. In the diabetes treatment context, for example, the control law delay can be interpreted as the time that elapses between the occurrence of a (food) disturbance and when it is actually reported to the controller — see Phan et al. (2016) for further discussion of the latter type of delay. In this context, it is important to note that the control law delay can actually be negative, i.e., when the occurrence of a disturbance is correctly anticipated. The results presented in the paper explicitly include the impact of both plant and control law delays on achievable performance.

The layout of the remainder of the paper is as follows: Section 2 describes the class of nonlinear positive systems used throughout the paper. Section 3 presents several preliminary properties of the model. Section 4 specifies signal constraints. Section 5 defines the underlying optimisation problem and the performance goal. Section 6 establishes the key result of the paper, namely that there exists a closed form solution to the aforementioned optimisation problem. Section 7 explains the relevance of the results to blood glucose regulation for Type 1 Diabetes. Section 8 presents a numerical example. Conclusions are presented in Section 9.

2. The model class

This section describes the model class. The key elements of the model are: (i) a nonlinear subsystem which includes a positive non-decreasing function, and (ii) a linear subsystem which is a stable all-real-pole system. In addition, all signals are positive and the manipulated input acts contrary to an external disturbance.

Remark 1. This problem formulation is consistent with the diabetes treatment problem in which insulin flow can only be positive and where food ingestion causes a positive transient in blood glucose. □

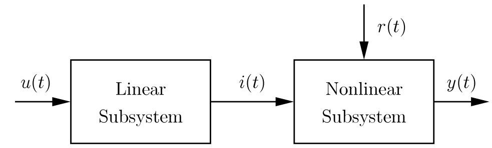

Fig. 1. System diagram.

The model is shown schematically in Fig. 1. Here $u(t)$, $i(t)$, $r(t)$ and $y(t)$ denote respectively the input, the input to the nonlinear subsystem, a disturbance and the output.

The signals $u(t)$, $i(t)$ are the input and the output of a linear subsystem. For convenience, these variables will also be converted to their corresponding incremental forms as follows:

$$u(t) = u_{ss} + \tilde{u}(t) \tag{1}$$

$$i(t) = i_{ss} + \tilde{i}(t) \tag{2}$$

where u_{ss}, i_{ss} denote steady state values, and $\tilde{u}(t)$, $\tilde{i}(t)$ denote increments from the steady state. Correspondingly, we denote

$$y(t) = y_{ss} + \tilde{y}(t). \tag{3}$$

2.1. Linear subsystem

The linear block denoted "Linear Subsystem" in Fig. 1 is assumed to be a stable all-real-pole linear system plus a plant delay denoted θ_p. We write

$$i(t) = \int_{-\infty}^{t} h(t-\tau)u(\tau-\theta_p)d\tau, \tag{4}$$

where $h(t)$ is the associated impulse response of the stable all-real-pole system and $\theta_p \geq 0$.

Stable all-real-pole linear systems often arise in models for systems described as a series of compartments (Anderson, 1983; Godfrey, 1983).

2.2. Nonlinear subsystem

The nonlinear subsystem in Fig. 1, is modelled as follows:

$$\dot{y}(t) = -(\varepsilon + m[i(t)])y(t) + r(t) + c, \tag{5}$$

where $y(t)$, $r(t)$, ε, c denote respectively the output variable, a non-negative disturbance, and two positive constants. A key assumption used in the sequel is the following.

Assumption 2. The function $m[i(t)]$ is a positive non-decreasing function. □

Remark 3. A special case of a function in this class is the Michaelis–Menten function (Menten & Michaelis, 1913), defined by

$$m[i(t)] = m^*[i(t)] = \frac{i(t)}{1+\beta \cdot i(t)}, \quad \beta \geq 0. \tag{6}$$

Such functions often appear in the modelling of biological and biochemical systems (Johnson & Goody, 2011). □

Remark 4. In the case $\beta = 0$, Eq. (5) becomes a bilinear model in which a product occurs between $i(t)$ and $y(t)$. □

Remark 5. The case $\beta = 0$ corresponds exactly to a model often used in numerical studies of the diabetes problem — see for example Bergman (2005) and Kanderian, Weinzimer, Voskanyan, and Steil (2009). □