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Dissipativity reinforcement in interconnected systems[☆]

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ABSTRACT

This paper focuses on the reinforcement of the quantitative performance in interconnected dynamical systems. The following problem is addressed that concerns dissipativity reinforcement via interconnection: Find a class of subsystems and their interconnection rule such that the L_2 gain bound of the entire interconnected system is reduced compared with that of each individual subsystem. We assume that each subsystem has a special passivity property that is characterized by two parameters, and has a bounded L_2 gain. Then, the feedback connection and the more general interconnection of the subsystems are expressed by the transition of the two parameters inheriting the same passivity property. In addition, the L_2 gain bound of the entire interconnected system, estimated with the parameters, is strictly reduced and becomes less than that of each subsystem. Finally, special interconnection rules are considered to show that the scale-expansion of the interconnected system, i.e., increasing the number of subsystems, gradually reduces the L_2 gain bound.

1. Introduction

A large-scale interconnected system is constructed from a number of subsystems that are internally connected with a specific rule. The analysis and design of such systems have been investigated in many studies by Antsaklis et al. (2013), Arcak and Sontag (2008), Bai, Arcak, and Wen (2011), Goodwine and Antsaklis (2013), Moylan and Hill (1978), Šiljak (1991), Stoustrup (2009) and Tan and Ikeda (1990). One motivating application is next generation power networks, which can be composed of a large number of renewable energy resources, such as solar, wind, and thermal generators. Because such generators are connected to an existing baseline network one after another, the entire network system is gradually built up and expanded in scale. At any stage of the expansion, the entire system should be stable and achieve a high performance in suppressing disturbances occurring in renewable energy resources. A final goal of this paper is to develop a systematic method for design and analysis of general expanding systems.

System design and control problems concerning general large-scale and expanding systems have previously been studied. For example, the studies by Antsaklis et al. (2013), Goodwine and Antsaklis (2013), Stoustrup (2009) and Tan and Ikeda (1990) have addressed the problems of stability analysis and stabilization of such expanding systems. They propose various design and control concepts, namely expanding construction by Tan and Ikeda (1990), plug and play control by Stoustrup (2009), and compositional stabilization by Antsaklis et al. (2013) and Goodwine and Antsaklis (2013). In addition, the passivity theorem (Zames, 1966) is utilized for the stabilization of interconnected systems. See, for example, the works by Arcak, Meissen, and Packard (2016), Bai et al. (2011), Hatanaka, Chopra, Fujita, and Spong (2015) and Moylan and Hill (1978). In the passivity-based design and analysis, the entire interconnected system inherits the passivity of the subsystems if they are connected according to a specific rule. We note that such conventional works aim *not to impair* the stability or *not to deteriorate* the performance of the entire system via the interconnection or scale-expansion, i.e., the increase of the number of subsystems. The aim of this paper is to determine a design strategy such that the performance of the entire system is *strictly and gradually improved* via the interconnection and scale-expansion.

In this paper, the following problem concerning *dissipativity reinforcement* is formulated for a large-scale interconnected system: Find a class of subsystems and their interconnection rule such that the L_2 gain bound of the entire interconnected system is *reduced* compared with that of each individual subsystem. We assume that each subsystem has a special passivity property that is characterized by two parameters, and has a bounded L_2 gain. Then, the feedback connection and the more general interconnection of the subsystems are expressed by the transition of the two

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parameters inheriting the same passivity property. In addition, the L_2 gain bound of the entire interconnected system, estimated with the parameters, is strictly reduced via the interconnection and its scale-expansion.

In the previous work by the present authors (Urata & Inoue, 2016, 2018), a special class of dissipative systems has been proposed. Problems regarding dissipativity reinforcement via feedback and other special interconnections have been formulated, and their solutions are presented by Urata and Inoue (2016, 2018). It has been shown that interconnected systems composed of special dissipative subsystems reduce the L_2 gain bound compared with each individual subsystem. In this paper, the result concerning the feedback connection is first further refined and generalized. Details of this generalization are provided in Section 3.3 of this paper. Next, a more general interconnection than the special one studied by Urata and Inoue (2016) is considered. Then, a general condition for dissipativity reinforcement is derived and a quantitative evaluation of the L_2 gain bound is presented. The analysis and evaluations are given in Section 4.

Notation: $\mathbb{R}_+ := [0, \infty)$. The symbol $\mathbf{1}_{k,\ell}$ represents a $k \times \ell$ matrix where every element is equal to one (this is said to be an all-ones matrix). The symbol $\mathcal{C}(c, r)$ represents a disk on the complex plane whose center and radius are given by $(c, 0)$ and r, respectively:

$$\mathcal{C}(c, r) := \{x + yi \in \mathbb{C} \mid (x - c)^2 + y^2 \le r^2\}.$$

The symbols L_2 and L_{2e} denote the L_2 space and the extended L_2 space, respectively. Let $\|\cdot\|$ be the Euclidean norm of a vector. Then, for $v \in L_2$ the symbol $\|v\|_{L_2}$ denotes the L_2 norm. For $v \in L_{2e}$ and $T \in \mathbb{R}_+$, the symbol $\|v\|_{L_2,T}$ denotes the finite time L_2 norm:

$$\|v\|_{L_2,T} := \left(\int_0^T \|v(\tau)\|^2 \mathrm{d}\tau\right)^{\frac{1}{2}}.$$

For a causal and L_2-stable system Σ, the symbol $\|\Sigma\|_{L_2}$ denotes the L_2 gain.

2. Preliminaries: system description and definition of dissipativity

2.1. Feedback and general interconnected systems

In this paper, we consider a feedback system $\Sigma_{\rm FB}$, and a more general interconnected system $\Sigma_{\rm NW}$, which are illustrated in Fig. 1. They are composed of two subsystems Σ_i, $i \in \{1, 2\}$, and of N subsystems Σ_i, $i \in \{1, 2, \ldots, N\}$, respectively. A subsystem Σ_i is a single-input-single-output (SISO) system, described as

$$\Sigma_i : \ y_i = \bar{\Sigma}_i u_i,$$

where $\bar{\Sigma}_i : L_{2e} \to L_{2e}$ is a causal operator, and $y_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ denote the output and input of Σ_i, respectively. If Σ_i is a linear time-invariant (LTI) system, then it can be described by the transfer function representation as

$$\Sigma_i : \ \bar{Y}_i(s) = \bar{\Sigma}_i(s)\bar{U}_i(s),$$

where $\bar{\Sigma}_i \in \mathbb{C} \to \mathbb{C}$ is the transfer function of Σ_i, and $\bar{Y}_i$ and $\bar{U}_i$ are the Laplace transformations of y_i and u_i, respectively. Although only the SISO system is studied in this paper, the results can be extended to multiple-input-multiple-output systems.

The negative feedback system $\Sigma_{\rm FB}$ is defined as follows. Let $w \in \mathbb{R}$ and $z \in \mathbb{R}$ be the external input and control output of $\Sigma_{\rm FB}$, respectively. Then, $\Sigma_{\rm FB}$ is constructed by the negative feedback connection as

$$u_1 = w - y_2 \tag{1}$$

$$u_2 = y_1 = z. \tag{2}$$

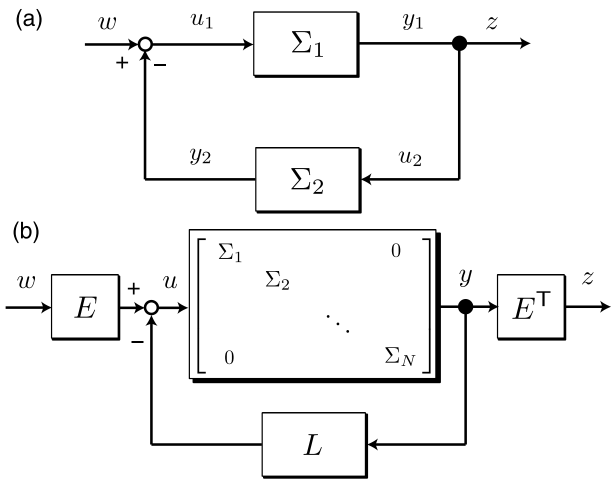

Fig. 1. Feedback system $\Sigma_{\rm FB}$ and general interconnected system $\Sigma_{\rm NW}$. Two dynamical systems Σ_1 and Σ_2 constitute a feedback system as illustrated in Fig. (a). Dynamical systems Σ_i, $i \in \{1, 2, \ldots, N\}$ are internally connected to each other according to a specified rule. The constructed network system is expressed as the feedback form as illustrated in Fig. (b).

The general interconnected system $\Sigma_{\rm NW}$ is defined as follows. In the same manner as the description of $\Sigma_{\rm FB}$, let w and z be the external input and control output of $\Sigma_{\rm NW}$, respectively. Further, define $u := [\, u_1\ u_2\ \cdots\ u_N\,]^{\mathsf T}$ and $y := [\, y_1\ y_2\ \cdots\ y_N\,]^{\mathsf T}$. Then, the interconnection rule in Σ_i, $i \in \{1, 2, \ldots, N\}$ is given as

$$u = Ew - Ly, \tag{3}$$

where $E \in \mathbb{R}^{N\times 1}$ and $L \in \mathbb{R}^{N\times N}$ are constant matrices. Here, E assigns the input port of w, while L represents a rule for the interconnection of Σ_i, $i \in \{1, 2, \ldots, N\}$. We suppose that z is given by

$$z = E^{\mathsf T} y, \tag{4}$$

which is utilized for the performance evaluation of $\Sigma_{\rm NW}$. We assume that E is of full column rank, and satisfies

$$E^{\mathsf T}E = 1.$$

In other words, E is normalized. In addition, we assume further that

$$E^{\mathsf T}L \ne 0, \quad LE \ne 0$$

holds. If this does not hold, then the effect of w cannot be controlled by the feedback input Ly or the information of z cannot be included in the feedback input. In this sense, this is a necessary assumption for the dissipativity analysis of $\Sigma_{\rm NW}$.

In this paper, we assume that $\Sigma_{\rm FB}$ and $\Sigma_{\rm NW}$ are well-posed, i.e., for any $w \in L_{2e}$, there exist unique solutions y_1 and y_2 of $\Sigma_{\rm FB}$ and y of $\Sigma_{\rm NW}$ that belong to L_{2e}.

2.2. Dissipativity

In order to quantitatively analyze Σ_i, $\Sigma_{\rm FB}$, and $\Sigma_{\rm NW}$, dissipativity (e.g. Hill and Moylan (1976); Willems (1972)) is defined and studied in this subsection.

There have been various definitions proposed for dissipativity (see, e.g. the book by Brogliato, Lozano, Maschke, Egeland, et al. (2006)). The dissipativity considered in this paper is defined as an integral quadratic constraint (IQC) that is specialized from the definition by Hill and Moylan (1976). For a matrix $\Pi \in \mathbb{R}^{2\times 2}$, consider

$$s(\Pi, u_i, y_i) := \begin{bmatrix} u_i \\ y_i \end{bmatrix}^{\mathsf T} \Pi \begin{bmatrix} u_i \\ y_i \end{bmatrix}.$$