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# A distributed economic MPC framework for cooperative control under conflicting objectives☆

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In this paper, we consider the problem of coordinating self-interested interacting dynamical systems by means of a distributed economic MPC framework. The self-interest of the systems is reflected by an individual local objective function each agent is trying to minimize, while cooperation is required with respect to coupling constraints and an asymptotic cooperative goal, which is represented by a particular steady state of the overall system. Our basic premise is that this steady state, which fulfills the cooperative goal, is not known a priori but has to be negotiated online, while already taking control actions. For the purpose of determining this steady state in a distributed way, arbitrary distributed computation algorithms can be incorporated into the proposed framework. We show that satisfaction of coupling constraints and convergence to the desired overall steady state can be established. Examples for an asymptotic cooperative goal include synchronization under conflicting objectives or sensor coverage, which are both studied in the work at hand and are also illustrated by numerical simulations.



## 1. Introduction

In recent years, networked systems have gained more and more attention in the control community. Advances in (wireless) communication technology result in increasingly complex interconnected systems such as groups of vehicles or electric networks, which in turn require specific control algorithms to coordinate them. Model predictive control (MPC) is an appealing control technique for such complex problems, since it ensures satisfaction of certain constraints and incorporates some performance criterion in terms of a cost function. However, applying a centralized model predictive controller to a distributed system is often not possible due to the large scale of the system and information not being available to a central unit. Besides, relying on a single central unit is error-prone and restricts scalability of the system. To overcome these issues, a large number of distributed MPC schemes have been proposed, see Christofides, Scattolini, de la Pena, and Liu (2013), Maestre and Negenborn (2013), Müller and Allgöwer (2017) and Scattolini (2009) for an overview. Many applications such as distributed power generation, multi-vehicle coordination or cooperating robot systems consist of a setup of uncoupled system dynamics but possibly coupling constraints in the state and input variables of the separate systems. Various distributed MPC algorithms have been proposed that achieve setpoint stabilization in such scenarios, for example in Dunbar and Murray (2006) with additional consistency constraints, in Grüne and Worthmann (2012) and Richards and How (2007) through the use of a certain sequential optimization algorithm, and in Spudic and Baotic (2013) via explicit MPC techniques. In Keviczky and Johansson (2008) and Müller, Reble, and Allgöwer (2012), distributed MPC algorithms were proposed for more general control problems such as consensus and synchronization. All of these mentioned algorithms are formulated in the framework of tracking MPC, in which the cost function is positive definite with respect to the specific setpoint or set to be stabilized.

However, stabilization of some setpoint might not be the primary control objective, but rather optimal operation with respect to a real performance criterion such as working costs or profit, which are in general not positive definite with respect to some steady state. For this reason, a more general MPC framework termed economic MPC was proposed in Angeli, Amrit, and Rawlings (2012), which allows for an arbitrary (economic) cost function that is not constrained to be positive definite with respect to the

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steady state. Unlike in tracking MPC, the economic MPC closed-loop system does not necessarily converge to a steady state, but there are numerous results on properties such as performance or fulfillment of average constraints, and under certain dissipativity properties of the system also stability and convergence results for this MPC framework available (see, e.g. Angeli et al., 2012, Grüne, 2013, Müller, Angeli, Allgöwer, Amrit, & Rawlings, 2014). Applying economic MPC in a distributed setup was proposed in Driessen, Hermans, and van den Bosch (2012) and Lee and Angeli (2014), where the overall steady state was assumed to be known and used as a terminal constraint.

The contribution of this work is a distributed economic MPC scheme for coordination of interacting systems under conflicting economic objectives. This antagonism between the requirement to achieve some desired coordinated behavior of the overall system and the self-interest of the individual systems of minimizing their individual running cost – the conflicting economic objective – will be the main theme of this work. More specifically, as for the coordination requirements we consider both transient coordination requirements, which are expressed in terms of coupling constraints on the system states and inputs and have to be satisfied at all times, and an asymptotic coordination requirement, which requires the systems to ultimately converge to a particular steady state of the overall system representing a certain cooperative goal. In contrast to Driessen et al. (2012) and Lee and Angeli (2014), our basic assumption is that this particular overall steady state is *not* known a priori but has to be negotiated between the systems online. Additionally, we assume that communication between systems is not instantaneous but requires time, which is why the systems already have to perform control actions while still negotiating the overall steady state satisfying the cooperative goal. The presented approach covers a large class of problems involving the coordination of self-interested systems. Coordination requirements that fit the proposed framework include, e.g., connectivity maintenance as transient coordination requirement, and asymptotic coordination requirements such as cost-optimal synchronization under conflicting objectives or sensor coverage. To the best of the author's knowledge, the proposed approach uniquely features distributed model predictive control subject to local, economic cost criteria while at the same time negotiating and eventually converging to an initially unknown overall steady state, which fulfills some asymptotic cooperative goal.

Some of the results of this paper have appeared in the conference versions (Köhler, Müller, & Allgöwer, 2016; Müller & Allgöwer, 2014). In the work at hand, we give a more comprehensive and holistic exposition of the proposed framework and its facets, including all proofs that have partly been left out in the conference versions. Moreover, we present the framework in a slightly generalized but much more flexible form. In particular, as opposed to Köhler et al. (2016) and Müller and Allgöwer (2014), in this work we allow for an arbitrary distributed coordination algorithm (e.g. distributed optimization algorithms or distributed controllers from classical multi-agent control) to be employed for online negotiation of an overall steady state satisfying an arbitrary cooperative goal. As such, the proposed framework allows to combine the capabilities of economic MPC for actual control of the individual systems with numerous and well-established tools from distributed optimization and multi-agent control, which is demonstrated in two example applications in Section 5.

The remainder of this paper is structured as follows. In Section 2, a detailed system description and problem statement are given. The proposed distributed economic MPC algorithm in its full generality is presented and analyzed in Section 3. This algorithm simplifies enormously when no transient coupling constraints are considered, which is why we treat this special case in Section 4 separately. Two example applications (sensor coverage and optimal synchronization) including numerical simulations are presented in Section 5, and some concluding remarks are given in Section 6.

## 2. Preliminaries and setup

### 2.1. Notation

For a set $\mathcal{A} \subseteq \mathbb{R}^n$ and a point $x \in \mathbb{R}^n$, the distance between the point $x$ and the set $\mathcal{A}$ is defined as $|x|_{\mathcal{A}} := \inf_{z\in\mathcal{A}}|x-z|$. For sets $\mathcal{A}, \mathcal{B} \subseteq \mathbb{R}^n$, the Minkowski set addition and set difference are defined by $\mathcal{A}\oplus\mathcal{B} := \{a+b \mid a \in \mathcal{A}, b \in \mathcal{B}\}$, and $\mathcal{A}\ominus\mathcal{B} := \{x \in \mathbb{R}^n \mid \{x\}\oplus\mathcal{B} \subseteq \mathcal{A}\}$, respectively. Let $\mathbb{I}_{\geq 0}$ denote the set of all nonnegative integers and $\mathbb{I}_{[a,b]}$ the set of all integers in the interval $[a, b] \subset \mathbb{R}$. The unit ball in $\mathbb{R}^n$ is denoted by $B_1$, i.e., $B_1 := \{x \in \mathbb{R}^n \mid |x| \leq 1\}$. A bounded sequence $v : \mathbb{I}_{\geq 0} \to \mathbb{R}^{n_v}$ is said to be *essentially convergent* to $\overline{v} \in \mathbb{R}^{n_v}$ if

$$\forall\varepsilon > 0 : \limsup_{T\to+\infty} \frac{\text{card}(\{0 \leq t \leq T : |v(t)-\overline{v}| \geq \varepsilon\})}{T+1} = 0.$$

Furthermore, as in Angeli et al. (2012), the set of asymptotic averages of $v$ is defined as

$$Av[v] := \{\bar{v} \in \mathbb{R}^{n_v} : \exists t_n \to +\infty : \lim_{n\to\infty} \frac{\sum_{k=0}^{t_n} v(k)}{t_n+1} = \bar{v}\}.$$

Note that $Av[v]$ is nonempty (as bounded sequences in $\mathbb{R}^{n_v}$ have limit points), but it need not be a singleton in general. We use undirected graphs to represent different kinds of interconnections between agents. A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of vertices $\mathcal{V} = \{1, \dots, P\}$ representing systems and edges $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V}\}$ representing interconnections between systems. Two systems $i, j$ are called *neighbors* if they are connected by an edge, i.e, if $(i, j) \in \mathcal{E}$. Denote the set of all neighbors of system $i$ by $\mathcal{N}_i(\mathcal{G}) := \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}\}$. A graph $\mathcal{G}$ is called *complete* if any two vertices are neighbors.

### 2.2. System description and problem statement

Consider a network of $P$ dynamical systems. Each agent is modeled as a discrete-time linear system of the form

$$x_i(t+1) = A_ix_i(t) + B_iu_i(t), \quad x_i(0) = x_{i0}, \tag{1}$$

with $x_i(t) \in \mathbb{X}_i \subseteq \mathbb{R}^{n_i}$ and $u_i(t) \in \mathbb{U}_i \subseteq \mathbb{R}^{m_i}$ for all $t \in \mathbb{I}_{\geq 0}$, and the pair $(A_i, B_i)$ is assumed to be stabilizable. The local state and input constraints for each system $(x_i(t), u_i(t)) \in \mathbb{Z}_i \subseteq \mathbb{X}_i \times \mathbb{U}_i$ are assumed to be compact and convex. Denote by $\mathcal{S}_i$ the set of all steady states for each agent $i$, i.e., $\mathcal{S}_i = \{(x_i, u_i) \in \mathbb{X}_i \times \mathbb{U}_i \mid A_ix_i + B_iu_i = x_i\}$. For the overall system, the state and input variables are written in stacked form as $x(t) = [x_1(t)^\top, \dots, x_P(t)^\top]^\top \in \mathbb{R}^n$ and $u(t) = [u_1(t)^\top, \dots, u_P(t)^\top]^\top \in \mathbb{R}^m$, where $n = \sum_i^P n_i$ and $m = \sum_i^P m_i$. The corresponding dynamics result in $x(t+1) = Ax(t) + Bu(t)$, $x(0) = [x_{10}^\top, \dots, x_{P0}^\top]^\top$, $(x(t), u(t)) \in \mathbb{Z}$ with $A = \text{diag}(A_1, \dots, A_P)$, $B = \text{diag}(B_1, \dots, B_P)$, $\mathbb{Z} = \{(x, u) \in \mathbb{R}^n \times \mathbb{R}^m \mid (x_i, u_i) \in \mathbb{Z}_i \ \forall i \in \mathbb{I}_{[1,P]}\}$ and the set of steady states $\mathcal{S} = \{(x, u) \in \mathbb{R}^n \times \mathbb{R}^m \mid (x_i, u_i) \in \mathcal{S}_i \ \forall i \in \mathbb{I}_{[1,P]}\}$. Let for any such set of overall states and inputs $\mathcal{A} \subseteq \mathbb{R}^n \times \mathbb{R}^m$ the projection of the set $\mathcal{A}$ on $\mathbb{R}^n$ and $\mathbb{R}^m$ be denoted by $\mathcal{A}_{\mathbb{X}} \subseteq \mathbb{R}^n$ and $\mathcal{A}_{\mathbb{U}} \subseteq \mathbb{R}^m$, respectively, i.e., $\mathcal{A}_{\mathbb{X}} := \{x \in \mathbb{R}^n \mid \exists u \in \mathbb{R}^m \ (x, u) \in \mathcal{A}\}$ and $\mathcal{A}_{\mathbb{U}} := \{u \in \mathbb{R}^m \mid \exists x \in \mathbb{R}^n \ (x, u) \in \mathcal{A}\}$.

Each agent acts self-interested and aims at minimizing its individual local cost function $\ell_i : \mathbb{Z}_i \to \mathbb{R}$, which is assumed to be continuous. Cooperation between the agents is required with respect to transient coordination requirements and asymptotic coordination requirements representing a cooperative goal. The transient coordination requirements are formulated as pointwise-in-time coupling constraints $\mathcal{C} \subseteq \mathbb{R}^n \times \mathbb{R}^m$ on the states and inputs of the form $(x, u) \in \mathcal{C}$ and have to be satisfied at all times, i.e., $(x(t), u(t)) \in \mathcal{C}$ for all $t \in \mathbb{I}_{\geq 0}$, where the set $\mathcal{C}$ is assumed to be convex. The asymptotic cooperative goal is represented by