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Brief paper

Performance boundary output tracking for one-dimensional heat equation with boundary unmatched disturbance ☆

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ABSTRACT

In this paper, we consider boundary output tracking for a one-dimensional heat equation with external disturbance at the opposite end of the bar. First, an unknown input infinite-dimensional observer is designed and an estimate of disturbance is obtained from the observer. Second, with reference signal and estimate of disturbance, we design a servo system which has bounded solution given that the reference signal and its derivative are bounded. The output feedback boundary control is then designed by the states of servo system and observer. It is proved that the state of closed-loop system tracks the state of the servo system. As a result, the output tracking is included. Finally, some simulation results are presented to illustrate the effectiveness of the proposed scheme.

1. Introduction

Output regulation is most often the main objective for a control system, which has various applications in military, civil engineering and many other fields. For finite dimensional systems, there are many classic results presented in different contexts, see, for instance Callier & Desoer (1980), Davison (1976), Desoer & Lin (1985) and Francis, 1977, to name just a few. Some output regulation results have been extended to the systems described by partial differential equations (PDEs). For this regard, we refer to Byrnes, Lauko, Gilliam, & Shubov (2000), Hämäläinen & Pohjolainen (2010), Immonen (2007), Pisano, Orlov, & Usai (2011) and Rebarber & Weiss (2003) and the references therein. Generally speaking, output regulation for PDEs is a nontrivial generalization from finite dimensional counterpart. In most of situations, the external disturbance is supposed to be "almost known", which is, together with reference signal, generated from an exogenous model both for finite dimensional systems (Deutscher, 2015) and infinite dimensional ones (Hämäläinen & Pohjolainen, 2010). To the best of our knowledge, there are few results on output regulation for PDEs when disturbance dynamics is unavailable. In Pisano et al. (2011), the disturbance is supposed to be general but the control operator is bounded. A typical case is presented in Paunonen and Pohjolainen (2014) where both input and output operators are unbounded but the disturbance dynamics is supposed to be modeled. In this note, we consider output tracking problem for a heat equation with general external disturbance at one end and the control is actuated on the other end. In other words, the control operator is unbounded and the external disturbance is not from any exosystem.

The central idea here comes from the active disturbance rejection control (ADRC) which is another powerful tool in dealing with uncertain ODEs and PDEs. The observer in output regulation to estimate modeled disturbance is generalized to the extended state observer to estimate the general disturbance in ADRC, and the feedback control is designed based on the online cancellation of disturbance. This keeps the energy saving characteristics of output regulation with modeled disturbance yet extends the generality of the disturbance. The ADRC has been generalized to stabilization of PDEs in Guo and Jin (2013) where the disturbance is estimated by infinitely many ODEs with high gain. Recently in Feng and Guo (2017), a new kind of observer for stabilization of PDEs is designed to estimate disturbance, where no high gain is needed. This new observer is applied to a tracking problem for a one-dimensional wave equation with control at one end and disturbance at the other end in Zhang, Feng, and Chai (2016). Motivated from Feng & Guo (2017), in this note, we design a state observer which is actually an extended state observer for a heat equation to estimate both state and disturbance simultaneously. Another motivation comes from recent papers (Guo & Guo, 2016; Guo, Guo, & Jin, 2015; Guo, Shao,

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& Krstic, 2017) where output tracking problem for one dimensional wave system with anti-collocated harmonic external disturbance has been discussed.

Motivated by the paper aforementioned, in this note, we consider output tracking for a one-dimensional heat equation with disturbance entering from the left end and control is actuated at the right, which is governed by the following partial differential equation:

$$\begin{cases} u_t(x,t) = u_{xx}(x,t),\ x \in (0,1),\ t > 0, \\ u_x(0,t) = d(t), t \geq 0, \\ u_x(1,t) = U(t), t \geq 0, \\ u(x,0) = u_0(x), 0 \leq x \leq 1, \\ y_m(t) = u(0,t), t \geq 0, \\ y_c(t) = u(1,t), t \geq 0, \end{cases} \tag{1}$$

where and henceforth u_x denotes the derivative of $u(x,t)$ with respect to x and u_t the derivative with respect to t, $U(t)$ is the input, $y_m(t)$ is the measurement, $u_0(x)$ is the initial value, $d(t)$ is the external disturbance, and $y_c(t)$ is the output to be regulated but is not measured. Given reference signal $y_{ref}(t)$, our target is to design an output feedback control $U(t)$ so that $y_c(t) \to y_{ref}(t)$ as $t \to \infty$.

The physical meaning of the model (1) is clear. The heat flux at the left end is supposed to be unknown and we measure the temperature at this end. The control is designed at the right end in terms of the measured temperature on the left end and the reference signal so that the temperature at the right end stabilizes asymptotically at the level of given reference temperature.

We consider system (1) in the state space $\mathcal{H} = L^2(0,1)$ with the norm induced by the inner product given by

$$\langle f, g \rangle_{\mathcal{H}} = \int_0^1 f(x)\overline{g(x)}dx, \quad \forall f, g \in \mathcal{H}. \tag{2}$$

The following definition of "exactly observable" is Definition 6.1.1 of Tucsnak and Weiss (2009).

Definition 1.1. Let X and Y be complex Hilbert spaces which are identified with their duals. Suppose that $T(t)$ is a strongly continuous semigroup on X, with generator $A : D(A) \to X$. X_1 is $D(A)$ with the norm $\|z\|_1 = \|(\beta I - A)z\|$, where $\beta \in \rho(A)$ is fixed. Consider the differential equation

$$\begin{cases} \dot{z}(t) = Az(t), \\ y = Cz(t), \\ z(0) = z_0 \in X_1, \end{cases}$$

where $C \in L(X_1, Y)$ is an admissible observation operator for $T(t)$. Let $\tau > 0$, and let Ψ_τ be the output operator associated with (A, C), i.e.

$$(\Psi_\tau z_0)(t) = \begin{cases} CT(t)z_0, & \text{for } t \in [0,\tau], \\ 0, & \text{for } t > \tau. \end{cases}$$

The pair (A, C) is exactly observable in time τ if Ψ_τ is bounded from below:

$$\int_0^\tau \|CT(t)z_0\|_Y^2 dt \geq C_\tau \|z_0\|_X^2$$

for some $C_\tau > 0$ and all $z_0 \in X$.

Proposition 1.1. *System (1) is not exactly observable with $d(t) \equiv 0$.*

Proof. Define the system operator A by

$$\begin{cases} Af = f'', \forall f \in D(A), \\ D(A) = \{f \in H^2(0,1) | f'(0) = f'(1) = 0\}. \end{cases} \tag{3}$$

The eigenvalues λ_n and corresponding eigenfunctions ϕ_n of A are computed as follows:

$$\begin{cases} \lambda_0 = 0, \lambda_n = -n^2\pi^2, n = 1, 2, \ldots \\ \phi_0(x) = 1, \phi_n(x) = \sqrt{2}\cos(n\pi x), n = 1, 2, \ldots. \end{cases} \tag{4}$$

These eigenfunctions of A form an orthonormal basis for $\mathcal{H}$ and A generates a semigroup e^{At}. The solution of (1) with $U(t) = d(t) = 0$ can be written explicitly as

$$u(x,t) = \int_0^1 u_0(x)dx + \sum_{n=1}^\infty \langle u_0, \phi_n \rangle e^{\lambda_n t}\phi_n(x), \tag{5}$$

from which

$$u(0,t) = \int_0^1 u_0(x)dx + \sqrt{2}\sum_{n=1}^\infty \langle u_0, \phi_n \rangle e^{\lambda_n t}. \tag{6}$$

If we take $u_0(x) = \phi_n(x)$, we obtain

$$u(0,t) = \int_0^1 \phi_n(x)dx + \sum_{j=1}^\infty \langle \phi_n, \phi_j \rangle e^{\lambda_j t}\phi_j(1)$$

$$= \int_0^1 \sqrt{2}\cos(n\pi x)dx + \sqrt{2}e^{\lambda_n t} = \sqrt{2}e^{\lambda_n t},$$

and hence

$$\begin{aligned} &\int_0^\tau u^2(0,t)dt = 2\int_0^\tau e^{2\lambda_n t}dt \\ &= \frac{1}{n^2\pi^2}(1 - e^{-n^2\pi^2\tau}) \geq C_\tau \|u_0\|^2, \end{aligned} \tag{7}$$

which is not true for any $C_\tau > 0$. Therefore, system (1) is not exactly observable with $d(t) \equiv 0$. $\square$

Nevertheless, we are still able to design an observer to estimate both the state and disturbance simultaneously with $y_m(t) = u(0,t)$ only, which constitutes a significant contribution of this note.

We proceed as follows. In Section 2, we design an observer which can recover the state and disturbance simultaneously. A servo system is therefore designed based on estimation of the disturbance. In Section 3, we design an output feedback control. The servo system produces reference signal on the one hand and bounded state on the other hand so that the original system state converges to the state of the servo system. This guarantees that in the process of tracking the reference signal, the closed-loop system is uniformly bounded. The well-posedness and tracking result are proved in this section. Some simulation results are presented in Section 4 to illustrate the effectiveness of the controller.

2. Observer design and servomechanism

In this section, we design an unknown input state observer with $(U(t), y_m(t))$ for system (1). A servo system is also designed with the reference signal and the estimate of disturbance. First, motivated from Feng & Guo, 2017 and Guo & Jin (2015), we design an unknown input observer for system (1) which is not exactly observable yet detectable as

$$\begin{cases} \hat{u}_t(x,t) = \hat{u}_{xx}(x,t), \\ \hat{u}(0,t) = y_m(t), \\ \hat{u}_x(1,t) = U(t), \\ \hat{u}(x,0) = \hat{u}_0(x), \end{cases} \tag{8}$$

where $U(t)$ and $u(0,t)$ are input and output of system (1). Let $\tilde{u}(x,t) = \hat{u}(x,t) - u(x,t)$. Then $\tilde{u}(x,t)$ satisfies

$$\begin{cases} \tilde{u}_t(x,t) = \tilde{u}_{xx}(x,t), \\ \tilde{u}(0,t) = 0, \\ \tilde{u}_x(1,t) = 0, \\ \tilde{u}(x,0) = \tilde{u}_0(x) = \hat{u}_0(x) - u_0(x), \end{cases} \tag{9}$$

which is well-posed and exponentially stable in the state space $\mathcal{H}$. In fact, define the system operator $\tilde{A}$ of (9) as

$$\begin{cases} \tilde{A}f = f'', \forall f \in D(\tilde{A}), \\ D(\tilde{A}) = \{f \in H^2(0,1) | f(0) = f'(1) = 0\}. \end{cases} \tag{10}$$