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Modal consensus, synchronization and formation control with distributed endogenous internal models☆

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ABSTRACT

Considering a group of heterogeneous agents communicating over a network, this paper introduces the innovative concept of *Distributed Endogenous Internal Model* as the key tool for a novel approach to formation control, synchronization and (modal) consensus. The novel strategy yields a dramatic reduction in terms of required communications and computations: in fact, while the usual approach to the mentioned problems entails that each agent is endowed with an internal model of the dynamics specifying the desired collective motion, in the novel approach such dynamics is *distributed over the network* among the agents, and it is realized in an *endogenous* fashion, namely by a suitable interconnection among parts of the dynamics already possessed by the agents, through the local cooperation between each agent and its neighbors. To address the cases when the purely endogenous solution is not viable, the related problem of how to minimally augment the dynamics of the overall network of agents in such cases is also studied.

1. Introduction

An increasing number of advanced applications requires that a large group of, possibly heterogeneous, independent agents interact over a communication network and achieves coordination in such a way to generate meaningful collective behaviors. Typical examples arise in several contexts, including biology, social sciences and engineering, and range from the simple requirement of having all the (physical or virtual) agents achieving consensus on the value of a variable of interest, hence implementing numerical distributed algorithms (Bürger, Notarstefano, Bullo, & Allgöwer, 2012; Li, Oikonomou, Tryfonas, Chen, & Xu, 2014; Sarlette, 2011; Tsitsiklis, Bertsekas, & Athans, 1986), to complex tasks requiring each agent to perform intricate maneuvers meanwhile moving in formation with neighboring agents; for a number of applications involving mobile robots, unmanned air vehicles (UAVs) and autonomous underwater vehicles, see *e.g.* Bullo, Cortes, and Martinez (2009), Fax and Murray (2004), Jadbabaie, Lin, and Morse (2003), Lin, Francis, and Maggiore (2005), Moreau (2005) and Olfati-Saber and Murray (2004). In a leader–follower context, the strategy in Sun and Cassandras (2016) guarantees that a formation is shaped and maintained in order to maximize a desired cost function, while preserving the connectivity of the team, whereas such additional objective is related to a *monitoring* task in Lin and Cassandras (2015). Moreover, the extension of consensus algorithms to manifolds that are not homeomorphic to an Euclidean space has been dealt with in Sarlette and Sepulchre (2009). An alternative source of nonlinearities in consensus problems, namely quantization of the communication channels, is considered in Carli, Fagnani, Frasca, and Zampieri (2010) and Frasca, Carli, Fagnani, and Zampieri (2009).

In addition to the challenges posed by the limited communications and available information, from a point of view closer to classic control theory it is clear that the possibility to achieve such coordinated motions is intrinsically related to the ability for each agent of the group to perform the individual motions required to obtain the desired common objectives. Since it turns out that such objectives may be naturally formulated within the framework of output regulation problems, the above problem can be indeed interpreted in terms of a distributed output regulation task for each agent, Su and Huang (2012); and it can be expected that the internal model principle (Francis & Wonham, 1976) plays a crucial role in achieving the required collective objective. This intuition essentially motivated several works, see *e.g.* De Persis and Jayawardhana (2014), Sepulchre, Paley, and Leonard (2008), Wieland, Sepulchre, and Allgöwer (2011) and Wieland, Wu, and Allgöwer (2013) and the references therein, that attacked the *consensus* problem – or, more in general, the problems related to formation control tasks – by ensuring that each agent in the group is provided with its own, complete *internal model* of the desired (common) behavior.

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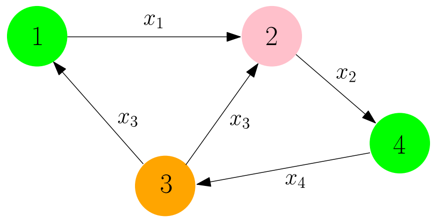

Fig. 1. A network of communicating heterogeneous agents, where arrows denote information flow, and homogeneous agents, in this case 1 and 4, are depicted in the same color. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

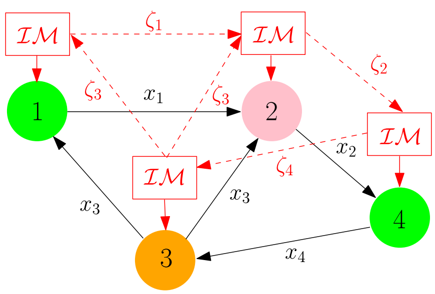

Fig. 2. The network in Fig. 1 with each agent endowed with its own internal model ($\mathcal{IM}$) dynamics; according to the classic approach, the state of each $\mathcal{IM}$ is exchanged to achieve synchronization among the $\mathcal{IM}$'s.

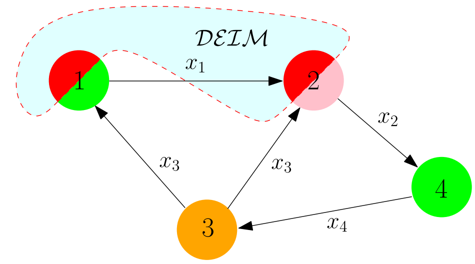

Fig. 3. The network in Fig. 1 where selected agents contribute part of their own dynamics to create a shared internal model for the whole network, so that a distributed endogenous internal model ($\mathcal{DEIM}$) is created.

Hence, considering for example the network of heterogeneous agents represented in Fig. 1 (where colors represent classes of homogeneous agents, so that different colors correspond to heterogeneous agents, and the allowed communications are shown by the arrows, which represent the admissible flows of information), the above mentioned approach consists in providing each agent in the group with its own complete copy $\mathcal{IM}$ of the desired dynamics, as can be seen in Fig. 2. In order to be useful for the considered task, such internal models $\mathcal{IM}$ need to be synchronized; for this purpose, consensus among the whole states of the internal models $\mathcal{IM}$ is enforced by standard Laplacian control laws, Wieland et al. (2011). Moreover, after synchronization, each agent has to achieve a suitable output regulation objective with respect to its own internal model, and then a regulator has to be designed, whose state dimension is also increasing with the complexity of the internal model $\mathcal{IM}$.

It is then evident that, in modern control applications involving the achievement of collective motions for extremely large-scale (even *huge*, in some circumstances) networks of agents, *e.g.* power grids (Dörfler & Bullo, 2012), data and sensor networks, Cortes, Martinez, and Bullo (2004) and Tsitsiklis and Bertsekas (1986), respectively, or the *Internet of Things* (Li et al., 2014; Xu, He, & Li, 2014), the replication of internal models depicted in Fig. 2, with the resulting increased load in terms of communications and computations, may turn out to be very costly, if not a limiting factor to the size of the solvable problem instances. Hence, implementing a solution that avoids replicating internal models in each individual node of the network may be a critical advantage as well as a *necessary step* towards the efficient exploitation of architectures consisting of large-scale networks of simple devices. This paper aims at addressing such issues by introducing *internal models that are essentially distributed over the entire network and are not, in general, entirely possessed by any agent of the group, and at the same time are realized by having a subset of agents share part of their states to generate a single internal model for the whole group*. This can be appreciated by comparing Fig. 3, which pictorially represents the *Distributed Endogenous Internal Model* ($\mathcal{DEIM}$), with Fig. 2; in the former, agents 1 and 2 contribute with a part of their dynamics and interactions to form the distributed internal model, and no additional dynamics is used (hence the internal model is endogenous).

The main contribution of this paper consists in introducing the concept of *distributed endogenous internal model* and in employing such notion to tackle the problems of modal consensus and formation control. The rest of the paper is organized as follows. In Section 2, we formally introduce the modal consensus and the formation control problems, together with interesting insights and notable variations. The novel notion of *distributed endogenous internal model* is the topic of Section 3. The solutions to the two problems defined above by means of distributed internal models are dealt with in Sections 4 and 5, respectively. Finally, numerical simulations, involving some interesting scenarios from the literature, are discussed in Section 6, while conclusions are drawn in Section 7. Preliminary results have appeared in Galeani and Sassano (2017). With respect to Galeani and Sassano (2017), herein we provide the characterization of the minimal dimension distributed internal model, necessary (in addition to sufficient) conditions for the existence of such models and the solution to the formation control problem with sequential deployment, together with the proofs of all the results and more detailed discussions and insights.

2. Preliminaries and problem definition

Consider a *multi-agent model* consisting of N heterogeneous linear systems described by equations of the form

$$\dot{x}_i = A_i x_i + B_i u_i \,, \tag{1a}$$
$$e_i = C_i x_i + Q_i w \,, \tag{1b}$$

with $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$ and $e_i(t) \in \mathbb{R}^{p_i}$, for $i = 1, \ldots, N$. Let $\bar{n} = \sum_{i=1}^{N} n_i$. The exogenous signal $w(t) \in \mathbb{R}^q$, which may be given several alternative interpretations as extensively discussed in the following, is generated by equations of the form

$$\dot{w} = Sw \,. \tag{2}$$

Assumption 2.1. The matrix S is such that[1] $\sigma(S) \subset \mathbb{C}^+ \cup \mathbb{C}^0$. ∘

The communication topology is captured by means of a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with vertex set $\mathcal{V} = \{v_1, \ldots, v_N\}$, each vertex associated to a system of the form of (1), and arc set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The latter encodes the information exchange pattern, namely the ith agent receives information from the jth agent if and only if $\{v_j, v_i\} \in \mathcal{E}$. The *adjacency matrix* $\mathcal{A}$ associated to the graph $\mathcal{G}$ is constructed element-wise by letting $a_{ij} = 1$ if and only if there is an arc between v_j and v_i and $a_{ij} = 0$ otherwise. Moreover, the notation $\mathcal{N}_i$ defines the set of neighbors of the agent i. Since the notion of

[1] $\sigma(A)$ denotes the spectrum of the matrix A.