Automatica 96 (2018) 121–126

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Decentralized cooperative tracking subject to motion constraints[☆]

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ARTICLE INFO

Article history:
Received 23 May 2016
Received in revised form 29 December 2017
Accepted 15 June 2018

Keywords:
Decentralized tracking
Formation control
Rigidity
Motion constraints
Attitude control

ABSTRACT

This paper addresses the formation control problem, where three agents are tasked with moving an object cooperatively along a desired trajectory while also adjusting its posture to some desired attitudes, i.e. position and attitude tracking. Two decentralized control laws based on locally available information are proposed. The first control law maintains constant inter-agent distances over time, i.e. the formation of agents moves as a single rigid-body. The second control law relaxes this constraint by only maintaining similarity of the agent formation as a polygon in Euclidean space.

1. Introduction

Formation control of systems with diverse dynamics and various task requirements has been studied using a number of different approaches to control design. For example, recently, some graphical conditions for distributed formation control were given in Lin, Wang, Han, and Fu (2014); Formation control with incomplete information (Jafarian & Persis, 2015; Liu & Jiang, 2013), with uncertainty (Dong & Farrell, 2009), with time-varying formation (Dong, Yu, Shi, & Zhong, 2015; Moya, Espinosa, Chávez, Leica, & Camacho, 2016; Turpin, Michael, & Kumar, 2012) were also investigated. Problems closely related to formation control are consensus, swarm, cooperative target tracking and path following. For an extensive literature review, see Bai, Arcak, and Wen (2011), Oha, Park, and Ahnb (2015), Ren and Beard (2008) and Ren and Cao (2011).

Much of the existing research on formation control address the design of decentralized control laws that steer the considered systems to some stable formation. The problem of how to maintain and change the attitude of an already established formation by means of decentralized control has generally been given less focus than formation stabilization. A possible application is having multiple robots carry one object in a decentralized fashion. Cooperation in such tasks is both crucial and difficult since any movement caused by one robot will affect the others. If for example the object being carried is rigid, the control should be designed to keep the relative distances between each robot or else they risk dropping or deforming the object.

In this paper, we consider a decentralized control task for three agents to carry an object collectively. The agents are to move the object along a desired trajectory while adjusting its posture to some desired attitude, which should perform three tasks simultaneously: (a) The distance between the agents should satisfy some constraints; (b) The carried object should follow some desired position path; (c) The carried object should follow some desired attitude path.

The motion constraint (a) arises from the object being carried by agents cooperatively. The scenario of fixed contact points on the object indicates constant-distance constraints between the agents. Furthermore, in order to change the grasping points to avoid obstacle, we also consider a constraint that allows the structures formed by the agents to maintain similarity throughout the evolution. A systematic framework for studying formation motion feasibility was developed in Tabuada, Pappas, and Lima (2005). Here, in addition to the feasibility analysis, we further investigate the following trajectory and attitude tracking problems.

[☆] This work was supported by the National Natural Science Foundation of China under Grant Nos. 61473189, 61773255 and 11688101, the Natural Science Foundation of Shanghai (No. 17ZR1445200), and the Swedish Research Council (2013-05438). The material in this paper was partially presented at the Proceedings of the 10th World Congress on Intelligent Control and Automation, July 6-8, 2012, Beijing, China. This paper was recommended for publication in revised form by Associate Editor Bert Tanner under the direction of Editor Christos G. Cassandras.

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https://doi.org/10.1016/j.automatica.2018.06.037

In order to fulfill the trajectory tracking task (b), we add a leader to guide the agents along the desired trajectory. The leader-following approach has been used in various scenarios in the past, where the leader's velocity or acceleration is usually assumed to be known by the followers (Hong, Chen, & Bushnell, 2008; Liu & Jiang, 2013; Zhang, Liu, & Feng, 2015). In this paper, we design an observer based on measurements of relative positions to calculate the velocity of the leader, and then embed it in the translation input to follow the desired trajectory.

The attitude regulation (c) is related to the attitude control problem for a rigid body which has long been a benchmark problem in robotics and related areas, see *e.g.* Markley and Crassidis (2014), Murray, Li, and Sastry (1994), Siciliano and Khatib (2008) and Wen and Kreutz-Delgado (1991). Much of the early work on attitude control was carried out using minimal representations of orientations such as Euler angles, or global many-to-one representations such as unit quaternions. In this paper, we consider a global parametrization that is in a one-to-one correspondence with the rotation matrix representation of an orientation as in Bayadi and Banavar (2014), Chaturvedi, Sanyal, and McClamroch (2011) and Markdahl, Hoppe, Wang, and Hu (2017). Unlike existing work on attitude control where the rigid body is actuated directly, here actuation on the body is mediated through the agents. In comparison with the attitude coordination of multiple rigid bodies in Bai, Arcak, and Wen (2008), Hatanaka, Igarashi, Fujita, and Spong (2012) and Igarashi, Hatanaka, Fujita, and Spong (2009) among other works, we focus on the attitude tracking of the object carried by the three agents rather than the attitudes of the agent themselves. Roughly speaking, the formation shaped by the three agents is taken to represent the attitude of the carried object. It is soft rather than hard rigidity.

A closely related research area is the load transport by multiple robots. Experimental results controlling a team of mobile robots in 2-D space were presented in Antonelli, Arri-chiello, and Chiaverini (2009), which were designed and implemented in a centralized architecture. Bilateral teleoperation between a single master and multiple cooperative slave robots was considered in Lee and Spong (2005) and Rodríguez-Seda, Troy, Erignac, Murray, Stipanović, and Spong (2010), where the slaves tracked their own reference points based on the master robot to perform position tracking and formation control. Decentralized motion and force control were designed for multiple robots to cooperatively transport objects in Mellinger, Shomin, Michael, and Kumar (2013), Montemayor and Wen (2005), Sugar and Kumar (2002) and Sun and Mills (2002), where desired trajectory was assigned to each robot in the motion control, and there was no exchange of information between the robots.

In our approach, decentralized controllers are designed for the agents to estimate the leader's velocity, and there is no pre-designed reference velocity or trajectory specified for each agent to track. Since under many circumstances, the motion control and the force control can be designed separately, in this paper, we focus on the motion control problem and simplify the agent to be single-integrator as in Cao, Morse, Yu, Ander-son, and Das gupta (2011), Marina, Cao, and Jayawardhana (2015), Marina, Jayawardhana, and Cao (2016) and Mou, Cao, and Morse (2015). Unlike the controllers in the above mentioned references that stabilize a rigid formation to a desired shape, we present a design that keep the shape all the time, and further extend the constant translation and rotation tracking in Marina et al. (2016) to a time-varying tracking scenario.

2. Problem statement

Assume the rigid object has been carried by the three agents initially, and there are no constraints on the magnitude of the available controlling torques. This enables us to focus on the cooperation issue of the three agents, rather than the force distribution problem. Let $x_i \in \mathbb{R}^3$ be the position of the ith agent, the dynamic is modeled by a single integrator

$$\dot{x}_i = u_i, \quad i = 1, 2, 3, \tag{1}$$

where u_i is the control input.

Let $d(x_i, x_j) = \|x_i - x_j\|^2$ denote the distances between agent i and j. Two motion constrains are considered, one is the rigid motion constraints that the distance between agents are fixed during the process; the other is the similarity motion constraints that the structures formulated by the three agents are flexible but keep similarity. The similarity motion constraints provide capability for obstacle avoidance and navigation in clustered environments.

Definition 2.1. Rigid motion constraint can be written as $d(x_i, x_j) = c_{i,j}$ for $i, j = 1, 2, 3$, where the constants $c_{i,j} > 0$ are given by the initial conditions.

Definition 2.2. Similarity motion constraint can be written as $d(x_1, x_2) = r_1 d(x_2, x_3)$ and $d(x_2, x_3) = r_2 d(x_3, x_1)$, where the constants $r_1, r_2 > 0$ are fixed and determined by the initial conditions.

We assume that the three agents are non-collinear, that is the three positions x_1, x_2 and x_3 uniquely characterize a plane. Since the object should be carried by the three agents all the time, the position and attitude of the object can be completely determined by the states of the agents. Therefore, we define the center of the three agents as

$$x_c = \frac{x_1 + x_2 + x_3}{3}, \tag{2}$$

and define two unit vectors as

$$n = \frac{(x_1 - x_2) \times (x_2 - x_3)}{\|(x_1 - x_2) \times (x_2 - x_3)\|}, \tag{3}$$

$$l = \frac{(x_1 - x_2) - (x_3 - x_1)}{\|(x_1 - x_2) - (x_3 - x_1)\|}, \tag{4}$$

where × is the cross product. One can see that n is a normal vector of the plane, and $l = (x_1 - x_c)/\|x_1 - x_c\|$ is a vector in the plane.

3. Motion constraints

First, we introduce some notations. Let

$$\xi = x_1 - x_2, \ \eta = x_2 - x_3, \ \zeta = x_3 - x_1. \tag{5}$$

For a vector $a = [a_1, a_2, a_3]^T \in \mathbb{R}^3$, $S(a) \in \mathbb{R}^{3\times 3}$ is the skew-symmetric matrix generated by a vector $a = [a_1, a_2, a_3]^T \in \mathbb{R}^3$, *i.e.*

$$S(a) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \tag{6}$$

Note that $S(a)b$, with $b \in \mathbb{R}^3$, is an alternative notation for the cross product $a \times b$. This implies, among other things, that $S(a)$ is linear and $S(a)a = 0$. Moreover, two properties of cross products are $a^T(b \times c) = c^T(a \times b)$ and $a \times (b \times c) = b(a^T c) - c(a^T b)$.

3.1. Rigid motion constraint

In order to maintain the relative distances $d(x_i, x_j)$ at constant values throughout the evolution, $\dot{d}(x_i, x_j) = 0$ implies that the control of system (1) should satisfy

$$\xi^\tau(u_1 - u_2) = 0,$$
$$\eta^\tau(u_2 - u_3) = 0,$$
$$\zeta^\tau(u_3 - u_1) = 0,$$