Automatica 95 (2018) 197–206

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Gradient extremum seeking for static maps with actuation dynamics governed by diffusion PDEs☆

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ARTICLE INFO

Article history:
Received 6 March 2017
Received in revised form 18 April 2018
Accepted 22 April 2018

Keywords:
Extremum seeking
Adaptive control
Backstepping
Averaging
Infinite-dimensional systems

ABSTRACT

We design and analyze the scalar gradient extremum seeking control feedback for static maps with actuation dynamics governed by diffusion PDEs. Conceptually, a non-model based online optimization control scheme is paired with actuation dynamics which occur in chemistry, biology and economics. A learning-based adaptive control approach with known actuation dynamics is considered in this paper. In the design part, we first compensate the actuation dynamics in the dither signals. Secondly, we introduce an average-based actuation dynamics compensation controller via a backstepping transformation, which is fed by the perturbation-based gradient and Hessian estimates of the static map. The stability analysis of the error-dynamics is based on using Lyapunov's method and applying averaging for infinite-dimensional systems to capture the infinite-dimensional state of the actuator model. Local exponential convergence to a small neighborhood of the optimal point is proven and illustrated by numerical simulations.

1. Introduction

1.1. Background and state-of-the-art

Extremum seeking control (ESC) is a non-model based, real-time optimization approach in the field of adaptive control. It is used to determine and maintain the extremum of an unknown, perhaps nonlinear, output map of a (stable) dynamic or static system. The goal is to find the optimal input Θ^* of an unknown plant that drives the measurable system output y to its unknown optimal value y^*, using the perturbation method.

The idea of ESC was first published in the paper of Leblanc (1922) for maximizing power transfer to a tram car. The Russians (Kazakevich, 1943) intensively studied this concept in the 1940s, before it was introduced in the 1950s in the US by Draper and Li (1951). The first general stability proof of ESC for stable dynamic systems with unknown output maps was performed by Krstić and Wang (2000). Since then, ESC has seen renewed interest and additional theoretical work, e.g., by Ariyur and Krstić (2003), stochastic ESC by Manzie and Krstić (2009), Newton-based ESC by Ghaffari, Krstić, and Nešić (2012) or ESC for stabilization by Scheinker and Krstić (2016) can be found in literature. Moreover, ESC was successfully applied in diverse engineering systems, such as mobile robots, bioprocesses, combustion engines, brake system control and more, which is summarized together with the history of ESC by Tan, Moase, Manzie, Nešić, and Mareels (2010) and references therein.

On the other hand, actuation dynamics described by infinite-dimensional systems has been introduced in several applications. The representative example is a system under actuator delays, in which the delay propagation can be described by a first order hyperbolic partial differential equation (PDE). Compensation of the actuator and sensor delay has been developed in Krstic and Smyshlyaev (2008a) for linear systems and in Krstić (2010) for nonlinear systems via the infinite-dimensional backstepping method (Krstić & Smyshlyaev, 2008b). Recently, the first contribution of applying ESC to infinite-dimensional actuation dynamics was achieved by Oliveira, Krstic, and Tsubakino (2017) in the case of actuator delays with known delay time. A stability proof of the error-dynamics and the convergence to the extremum was presented.

Another well known infinite-dimensional system is a diffusion process which arises in several biological, chemical and economical systems (see Edelstein-Keshet (1988), Wang, Wang, Xu, Wu, and Jia (2013)). Compensation of actuator dynamics governed by a diffusion PDE was studied in Krstić (2009) for the stabilization of a linear system. However, there is no work in the literature which

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Thomas Meurer under the direction of Editor Daniel Liberzon.

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https://doi.org/10.1016/j.automatica.2018.05.023
0005-1098/

concerns ESC in the presence of actuation dynamics governed by diffusion PDEs, or more general parabolic PDEs.

Although our main focus is to consider more general dynamical systems rather than those governed by ordinary differential equations, where extremum seeking methodology can be extended to a class of diffusion PDEs, our problem setting is close to be applied to practical scenarios since it satisfies the basic conditions raised in chemical engineering literature. Motivated by examples like optimizing the product rate of a tubular bioreactor (Cougnon, Dochain, Guay, & Perrier, 2006; Hudon, Guay, Perrier, & Dochain, 2008; Winkin, Dochain, & Ligarius, 2000), where the model is described by coupled (in-domain) linear parabolic PDEs, we can consider problems with the dynamical model of the bioreactor as infinite-dimensional actuator dynamics, which generates an exponentially stable $\mathscr{C}_0$-semigroup, as shown in Winkin et al. (2000). Finding and maintaining the optimal unknown product rate, described by a static map, is a more advanced setting of the problem we consider in this paper. Therefore, we first study the pure diffusion actuation dynamics paired with ESC, before we extend the results to more complex applications.

We tackle this problem with a semi-model-based or so-called "partially model based" approach, since the diffusion coefficient is known, but the plant model (map) is unknown and its parameters are estimated using perturbations, as done in Oliveira et al. (2017). As previously mentioned by learning-based controllers in Benosman (2016) and Benosman, Lewis, and Guay (2018), such kind of controllers are partly based on a physics-based model, and partly based on a model-free learning algorithm. In this paradigm, model free learning is used to complement the physics-based model and compensate for the uncertain or the missing part of the model.

1.2. Contributions and organization of the paper

In this paper, we design ESC for static maps with actuation dynamics governed by diffusion PDEs. The controller to compensate the known actuation dynamics is designed via an infinite-dimensional backstepping transformation and is fed with the gradient and Hessian estimate of the static map. The main contribution of this paper is on stability analysis. Firstly, the transformed target system associated with the low-pass filtered boundary value for ESC is shown to be exponentially stable. Secondly, invoking the averaging theorem for infinite-dimensional systems was successfully applied to the parabolic PDE of the average system via semigroup analysis, while in Oliveira et al. (2017) the system was described as a functional differential equation. Finally, convergence to the neighborhood of the extremum is proven.

In the next section, we introduce the problem statement with setting up the mathematical formulations. Section 3 provides the design of the diffusion compensation controller and the derivation of the error-dynamics. Section 4 presents the proof of our main theorem on stability and convergence of the error-dynamics for ESC with actuation dynamics governed by diffusion PDEs. Numerical simulations are performed in Section 5, which validate the stability and convergence property of the closed-loop system. In Section 6 we give a conclusion of the paper.

1.3. Notation

$\mathscr{C}^n(\mathscr{X})$ denotes a n-times continuously differentiable function on the domain $\mathscr{X}$. $\mathbb{R}_+$ stands for the domain of positive real numbers including 0. We denote the partial derivatives of a function $u(x,t)$ as $\partial_x u(x,t) = \partial u(x,t)/\partial x$, $\partial_t u(x,t) = \partial u(x,t)/\partial t$. The 2-norm of a finite-dimensional (ODE) state vector $\vartheta : \mathbb{R}_+ \to \mathbb{R}$ is denoted by single bars, $|\vartheta(t)|$. In contrast, norms of functions (of x) are denoted by double bars. We denote the spatial $\mathscr{L}_2[0,D]$ norm of the PDE state $u(x,t)$ as $\|u(t)\|^2_{\mathscr{L}_2([0,D])} := \int_0^D u^2(x,t)dx$, where we drop the index $\mathscr{L}_2([0,D])$ in the following, hence $\|\cdot\| = \|\cdot\|_{\mathscr{L}_2([0,D])}$, if not otherwise specified. As defined in Khalil (1996), a vector function $f(t,\varepsilon) \in \mathbb{R}$ is said to be of order $\mathscr{O}(\varepsilon)$ over an interval $[t_1,t_2]$, if $\exists k, \bar{\varepsilon} : |f(t,\varepsilon)| \le k\varepsilon, \forall \varepsilon \in [0,\bar{\varepsilon}]$ and $\forall t \in [t_1,t_2]$. In most cases we give no estimation of the constants k and $\bar{\varepsilon}$, then $\mathscr{O}(\varepsilon)$ can be interpreted as an order of magnitude relation for sufficiently small ε.

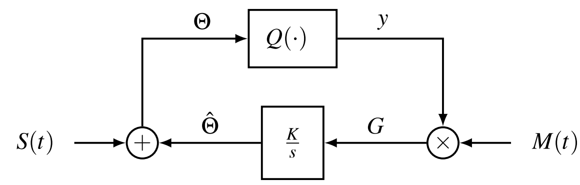

Fig. 1. Basic gradient extremum seeking scheme.

2. Problem statement

2.1. Basic gradient extremum seeking

As introduced, ESC for static maps is a real-time optimization control scheme, where the goal is to find and maintain the optimum of an unknown nonlinear static map $Q : \mathbb{R} \to \mathbb{R}$ with optimal unknown output $y^* \in \mathbb{R}$, unknown optimizer $\Theta^* \in \mathbb{R}$, measurable output $y \in \mathbb{R}$ and input $\Theta \in \mathbb{R}$ (see Fig. 1). Without loss of generality, we consider maximization problems (for minimization use $y = -y$). The method of sinusoidal perturbation (Khalil, 1996) forms the basics of extremum seeking (Ariyur & Krstić, 2003; Krstić & Wang, 2000), such that the input parameter Θ of the static map is varying to estimate the gradient G of the static map. Therefore, the perturbation signal

$$S(t) = a\sin(\omega t), \tag{1}$$

with perturbation amplitude a and frequency ω, is added to the estimation of the optimizer Θ^*, given by $\hat{\Theta}$. The dither signal to estimate the gradient of the static map is chosen as

$$M(t) = \frac{2}{a}\sin(\omega t). \tag{2}$$

The idea of choosing the dither signals as (1) and (2) is derived such that the averaged signal of the gradient estimate G is given, under the assumption of a quadratic map $Q(\cdot)$, by $G_{\mathrm{av}}(t) = H\tilde{\Theta}_{\mathrm{av}} = H(\hat{\Theta}_{\mathrm{av}} - \Theta^*)$, where H is the unknown negative Hessian of the static map and $\tilde{\Theta} = \hat{\Theta} - \Theta^*$ the estimation error. This yields the averaged error-dynamics $\dot{\tilde{\Theta}}_{\mathrm{av}} = KH\tilde{\Theta}_{\mathrm{av}}$, with adaption gain $K > 0$. Since $Q(\cdot)$ can be nonlinear, but approximated as quadratic map in a neighborhood of the optimum (Θ^*, y^*), the system in Fig. 1 is locally exponentially stable and by the averaging theorem in Khalil (1996), the local exponential stability of the original error-dynamics $\dot{\tilde{\Theta}} = KH\tilde{\Theta}$ follows.

2.2. Actuation dynamics and output

In addition to the basic ESC scheme in Fig. 1, we consider actuation dynamics which are described by a diffusion process, i.e., a heat equation with the actuator $\theta(t) \in \mathbb{R}$ and the propagated actuator $\Theta(t) \in \mathbb{R}$ given by

$$\Theta(t) = \alpha(0,t) \tag{3}$$

$$\partial_t \alpha(x,t) = \partial_{xx}\alpha(x,t), \quad x \in (0,D) \tag{4}$$

$$\partial_x \alpha(0,t) = 0 \tag{5}$$

$$\alpha(D,t) = \theta(t), \tag{6}$$