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Brief paper

Robust hierarchical model predictive control of graph-based power flow systems ☆

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ABSTRACT

A robust hierarchical model predictive control framework is presented for controlling a linear system of dynamically coupled subsystems. A graph-based modeling framework captures the conservation laws of power flow systems, for which control optimizes the storage and routing of energy to maximize transient and steady-state power throughput. A constructive approach is presented for developing an N-level hierarchical controller, which guarantees satisfaction of state and input constraints in the presence of signal and model uncertainty.

1. Introduction

Graph-based modeling approaches have been used in a wide variety of applications areas such as chemical processing plants (Jogwar, Rangarajan, & Daoutidis, 2015; Preisig, 2009), building thermal systems (Mukherjee, Mishra, & Wen, 2012), electronic circuits (Behjati, Davoudi, & Lewis, 2014), and flow control systems (Blanchini, Franco, Giordano, Mardanlou, & Montessoro, 2016). These systems function based on the storage, conversion, and routing of conserved quantities such as mass and energy and thus can be modeled as a graph; where vertices and edges represent the storage and transport, respectively. These systems typically have hard constraints bounding the actuators and states along with dynamics that evolve over a wide range of timescales.

A decentralized model predictive control (MPC) approach that guarantees closed-loop stability for graph-based power flow systems has been presented in Koeln and Alleyne (2017). Using the inherent passivity of these systems, a single passivity constraint is added to each of the decentralized controllers, making guaranteed stability a local and easily implementable modification to existing MPC frameworks. While stability is achieved, robust feasibility of state constraints is not guaranteed.

While considering the same general class of systems, the goal of this paper is to achieve robust feasibility of actuator and state constraints of discrete-time linear power flow systems under hierarchical MPC to optimize the storage and routing of conserved quantities across multiple timescales. These conserved quantities are referred to as the storage of *energy* and the routing of *power* without loss of generality. By achieving robust feasibility, the constrained optimization problem for each controller in the hierarchy remains feasible in the presence of model uncertainty and disturbance signal uncertainty. Additionally, by satisfying state and input constraints, bounded-input, bounded-output (BIBO) stability is guaranteed.

Building on a number of robust centralized (Langson, Chryssochoos, Raković, & Mayne, 2004; Limon, Alvarado, Alamo, & Camacho, 2008, 2010; Mayne, Seron, & Raković, 2005) and distributed (Farina & Scattolini, 2012; Riverso & Ferrari-Trecate, 2012; Trodden, 2014; Trodden & Richards, 2010) MPC formulations, several robust hierarchical MPC formulations have been developed. In Scattolini and Colaneri (2007), a two-level hierarchical control approach is presented with a slow higher-level and fast lower-level controller. The lower-level controller bounds deviations between the control decisions made at each level and the higher-level controller is made robust to these deviations using a min–max robust MPC formulation. This approach is extended in Scattolini, Colaneri, and Vito (2008) by allowing the lower-level of control to consist of m controllers for systems with decoupled actuator dynamics. The

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goal of the upper-level controller is to determine which actuators to enable along with their desired control inputs, while the lower-level controllers determine the actual control inputs that account for the dynamics of the actuators at a faster timescale. This work is further formalized in Picasso, Vito, Scattolini, and Colaneri (2010). Additional approaches are presented in Barcelli, Bemporad, and Ripaccioli (2010, 2011) and Vermillion, Menezes, and Kolmanovsky (2014) where two-level hierarchical controllers are developed that act similar to reference governors, using dynamic actuators to satisfy system constraints with guaranteed stability.

In each of these efforts, a two-level hierarchical framework is developed to handle the timescale separation between the system and actuator dynamics. However, in practice, many systems have more than two timescales and an N-level hierarchical controller would be more effective. While Picasso, Romani, and Scattolini (2009) present a more generic mathematical formulation for N-level hierarchical MPC, theoretical properties like robust stability and feasibility are not established and the authors state that "much work is still needed".

The main features of the proposed approach are: (1) the control hierarchy has N levels to match the N timescales of the system, (2) the system is decomposed into multiple subsystems to reduce computational cost of low level controllers, (3) the formulation guarantees state and actuator constraint satisfaction in the presence of both model and disturbance signal uncertainty, and (4) model reduction is employed to reduce computational costs of high level controllers. With these benefits, the proposed approach relies on several assumptions about the system and control formulation that are discussed throughout the paper. Relaxing these assumptions is the focus of ongoing work.

The remainder of the paper is organized as follows. Section 2 presents the class of graph-based power flow systems and Section 3 provides an overview of the hierarchical control structure. The MPC formulation for each controller in the hierarchy is presented in Section 4. Section 5 presents the main result of the paper establishing the recursive feasibility of all controllers in the hierarchy. A numerical example is presented in Section 6, demonstrating the hierarchical control formulation. Finally, concluding remarks are provided in Section 7.

Notation

For sets $\mathcal{X}, \mathcal{Y} \subset \mathbb{R}^n$, the Minkowski sum is $\mathcal{X} \oplus \mathcal{Y} \triangleq \{x + y \mid x \in \mathcal{X},\ y \in \mathcal{Y}\}$ and for sets $\mathcal{Y} \subset \mathcal{X}$, the Pontryagin difference is $\mathcal{X} \ominus \mathcal{Y} \triangleq \{x \in \mathbb{R}^n \mid \mathcal{Y} + x \subset \mathcal{X}\}$. For a set $\mathcal{X} \subset \mathbb{R}^n$ and the linear mapping $A : \mathbb{R}^n \to \mathbb{R}^m$, $A\mathcal{X} \triangleq \{Ax \mid x \in \mathcal{X}\}$. A set $\mathcal{X} \subset \mathbb{R}^n$ is robust positively invariant (RPI) for a system $x(k+1) = f(x(k), w(k))$ if and only if for all $x \in \mathcal{X}$ and all $w \in \mathcal{W}$ it holds that $f(x(k), w(k)) \in \mathcal{X}$. The right inverse of $A \in \mathbb{R}^{n\times m}$ is defined as $A^\dagger = A^T(AA^T)^{-1}$. The value of a variable x at time k is denoted $x(k)$ or simply x for notational simplicity. The value at $k+1$ is denoted x^+. The double notation $x(i|j)$ denotes the predicted value of x at future time i determined at time j.

2. Class of systems

Consider the power flow system **S** represented by an oriented graph $\mathcal{G} = (V, E)$ of order N^v with the set of vertices $V = \{v_i\}$, $i \in [1, N^v]$ and of size N^e with set of edges $E = \{e_j\}$, $j \in [1, N^e]$. Each oriented edge $e_j \in E$ represents power flow in **S**, where positive power P_j flows from the tail vertex v_j^{tail} to the head vertex v_j^{head}. Each vertex $v_i \in V$ has an associated state x_i that represents the energy stored in that vertex. Thus, the dynamic for the state of each vertex v_i satisfies the discrete-time energy conservation equation

$$C_i \frac{x_i^+ - x_i}{\Delta t} = \sum_{e_j \in E_i^{in}} P_j - \sum_{e_j \in E_i^{out}} P_j, \tag{1}$$

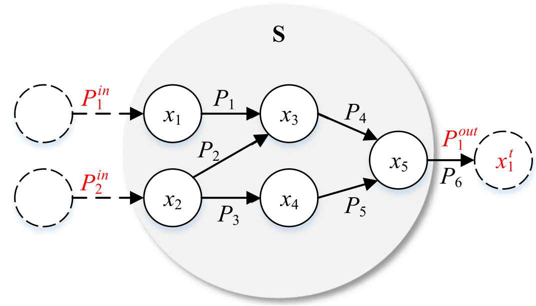

Fig. 1. Notional system exemplifying the graph-based power flow representation with key disturbances and power flows highlighted in red. Dashed lines indicate elements that serve as disturbances to the system. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

where Δt is the time step and $C_i > 0$ is the energy storage capacitance of vertex v_i while $E_i^{in} = \{e_j | v_j^{head} = v_i\}$ and $E_i^{out} = \{e_j | v_j^{tail} = v_i\}$ are the sets of edges oriented into and out of vertex v_i.

Assumption 1. The power flow P_j along edge e_j is defined as

$$P_j = a_j x_j^{tail} - b_j x_j^{head} + c_j u_j + \Delta P_j, \tag{2}$$

where x_j^{tail} and x_j^{head} are the states of the tail and head vertices v_j^{tail} and v_j^{head}, u_j is an associated actuator input, the coefficients $(a_j, b_j) \geq 0$ and $c_j \neq 0$ define the relationship between the power flow, the states, and the actuator input, and the disturbance $|\Delta P_j| \leq \Delta P_j^{max}$ is an unknown but bounded additive power flow.

Remark 1. In (2), ΔP_j is treated as an unknown, yet bounded, disturbance. This disturbance represents both model uncertainty and bounded linearization error when using (2) to approximate nonlinear power flow relationships.

In general, the system **S** has states $x \in \mathbb{R}^{N^v}$ that each satisfy (1) and power flows $P \in \mathbb{R}^{N^e}$ that each satisfy (2). The disturbances to **S** capture how power enters and exits the system, with inlet power flows $P^{in} \in \mathbb{R}^{N^s}$ and sink states $x^t \in \mathbb{R}^{N^t}$. As indicated by dashed lines in Fig. 1, the inlet power flow edges are not included in $\mathcal{G}$. Also indicated by dashed lines in Fig. 1, the sink states are *not* states of **S**, but the sink vertices and the edges connecting **S** to the sink vertices *are* included in $\mathcal{G}$. Power flows along this type of edge, denoted $P^{out} \in \mathbb{R}^{N^t}$, each follow the relationship from (2).

Let $M = [m_{i,j}] \in \mathbb{R}^{(N^v+N^t)\times N^e}$ be the incidence matrix of graph $\mathcal{G}$ (West, 2001) where

$$m_{i,j} = \begin{Bmatrix} +1 & \text{if } v_i \text{ is the tail of } e_j \\ -1 & \text{if } v_i \text{ is the head of } e_j \\ 0 & \text{else} \end{Bmatrix}. \tag{3}$$

Then, based on (1), the system dynamics are

$$\begin{bmatrix} C(x^+ - x) \\ (x^t)^+ - x^t \end{bmatrix} = -\Delta t M P + \Delta t \begin{bmatrix} D \\ 0 \end{bmatrix} P^{in}, \tag{4}$$

where $C = diag([C_i])$ is a diagonal matrix of the vertex capacitances and $D = [d_{i,j}] \in \mathbb{R}^{N^v\times N^s}$ where

$$d_{i,j} = \begin{Bmatrix} 1 & \text{if } v_i \text{ is the head of } P_j^{in} \\ 0 & \text{else} \end{Bmatrix}. \tag{5}$$

Since x^t are disturbances to the system, not states, M is partitioned as $M = \begin{bmatrix} \bar{M} \\ \underline{M} \end{bmatrix}$, with $\bar{M} \in \mathbb{R}^{N^v\times N^e}$ and $\underline{M} \in \mathbb{R}^{N^t\times N^e}$, resulting in

$$C(x^+ - x) = -\Delta t \bar{M} P + \Delta t D P^{in}. \tag{6}$$