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Zonotope-based recursive estimation of the feasible solution set for linear static systems with additive and multiplicative uncertainties ☆

Hao Wang [a,*], Ilya V. Kolmanovsky [b], Jing Sun [a]

[a] Department of Navel Architecture and Marine Engineering, University of Michigan, Ann Arbor, MI 48109, USA
[b] Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109, USA

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ABSTRACT

In this paper, we develop two zonotope-based set-membership estimation algorithms for identification of time-varying parameters in linear static models, where both additive and multiplicative uncertainties are treated explicitly. The two recursive algorithms can be differentiated by their ways of processing the data and required computations. The first algorithm, which is referred to as Cone And Zonotope Intersection (CAZI), requires solving linear programming problems at each iteration. The second algorithm, referred to as the Polyhedron And Zonotope Intersection (PAZI), involves linear programming as well as an optimization subject to linear matrix inequalities (LMIs). Both algorithms are capable of providing tight overbounds of the feasible solution set (FSS) in an application to health monitoring of marine engines. Furthermore, PAZI algorithm applied to mini-batches of measurement data leads itself to further analysis of the relation between the estimation results at different iterations. In addition, an example of identifying time-varying parameters is also reported.

1. Introduction

Set-membership estimation has been studied by many researchers since 1960s (Schweppe, 1968). This approach is often referred to as a guaranteed estimation approach and it generates typically an overbound of the feasible solution set (FSS), which consists of all possible parameters that are consistent with measurements, models, assumptions on noise, and uncertainty bounds. Unlike statistical estimation techniques, no assumptions are made in set-membership estimation about probability distribution of process noise and measurement noise. Recent developments in set-membership estimation include recursive algorithm development and comparison (Casini, Garulli, & Vicino, 2017a, 2017b), new insights revealing the connections with statistical estimation (Fernández-Cantí, Blesa, Puig, & Tornil-Sin, 2016; Wei, Liu, Song, & Liu, 2015), new techniques for handling nonlinearities (Fernández-Cantí, Tornil-Sin, Blesa, & Puig, 2015), applications to model reference control (Guo, Zhang, & Jiang, 2016; Rotondo, Nejjari, Puig, & Blesa, 2015) and robust MPC (Ping & Sun, 2015), and other novel applications (e.g. simultaneous localization and mapping (SLAM) Yu, Zamora, & Soria, 2016 and diabetes treatment Herrero et al., 2016). Set-membership techniques for simultaneous input and parameter estimation are developed in Kolmanovsky, Sivergina, and Sun (2006).

A major topic considered in set-membership estimation is the parameter identification in linear systems. As shown in Milanese and Vicino (1996), the FSS of unknown constant parameters can be computed exactly if a linear system with an additive uncertainty is considered. But solving this problem numerically is computationally very involved, thus the approximated feasible solution set (AFSS) is often sought as the over-approximation of the FSS (i.e. FSS $\subseteq$ AFSS). Commonly-used geometric elements for recursively performing such approximations are boxes (Casini et al., 2017a), ellipsoids (Kurzhanski & Valyi, 1997), and zonotopes (Bravo, Alamo, & Camacho, 2006; Le, Stoica, Alamo, Camacho, & Dumur, 2013). Recently, zonotopes have become popular (Le et al., 2013) as the procedures exploiting zonotopes have high computational efficiency and can provide tight overbounds of the FSS. In particular, in Bravo et al. (2006), a zonotope-based algorithm is developed for handling the problem of estimating time-varying parameters. This problem is revisited in this paper for the case with both additive and multiplicative uncertainties while in previous literature (Bravo et al., 2006; Casini et al., 2017a, 2017b; Le et al., 2013; Milanese & Vicino, 1996) only additive uncertainties were treated. The system studied throughout this paper is described as follows.

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* Corresponding author.
E-mail addresses: autowang@umich.edu (H. Wang), ilya@umich.edu (I.V. Kolmanovsky), jingsun@umich.edu (J. Sun).

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Consider a linear parametric model in the form treated in Bravo et al. (2006), Casini et al. (2017a), Le et al. (2013) and Milanese and Vicino (1996), and generalized to a Multiple-Measurement setting,

$$y_k = \phi_k^\top \theta_k + u_k, \tag{1}$$

where $y_k \in \mathbf{R}^m$ is the measured system output, $\theta_k \in \mathbf{R}^n$ represents the true parameter vector to be estimated, $\phi_k \in \mathbf{R}^{n\times m}$ is the regressor and $u_k \in \mathbf{R}^m$ represents the additive noise. When u_k in (1) is unknown but has known bounds, we refer to the uncertainty associated with u_k as "additive" because of the way it enters the parametric model in (1). There are well-established set-membership estimation algorithms for this case. However, in many cases, such as the engine condition monitoring problem addressed in Wang, Kolmanovsky, and Sun (2016) that motivated this study, the uncertainties may affect the regressor, entering Eq. (1) in a multiplicative form in relation to the unknown parameters. In Casini, Garulli, and Vicino (2014) and earlier literature (see Cerone, 1993; Kuntzevich & Lychak, 1992; Kuntzevich, Lychak, & Nikitenko, 1988; Norton, 1987), it is shown that FSS in a problem with constant unknown parameters is, in general, non-convex. A convex relaxation approach can be pursued with box-type solutions for FSS, which may lead to very conservative over-approximations in some cases. A convex relaxation approach to a similar problem is pursued in Cerone, Piga, and Regruto (2012), where the problem involving multiplicative uncertainties is referred as an Error-in-Variable (EIV) problem. The setting in Cerone et al. (2012) is different from ours in that Cerone et al. (2012) explicitly handles the dependence of the regressor on past outputs while in our health monitoring applications this dependence does not appear, and hence we focus on the latter case, which may be referred to as set-membership identification of linear static systems. When applied in a setting of Cerone et al. (2012), our algorithms may provide more conservative results as they do not use extra modeling information. In addition, we further assume that the true parameters to be positive. We note that the positiveness of the parameters and the independence of the regressor from the past system outputs are restrictive with respect to solving the general set-membership problem for linear systems affected by bounded errors-in-variables. However, for several practical applications including the marine engine health monitoring considered in this paper, these assumptions are reasonable. In engine health monitoring problem, parameters to be estimated are positive, since they represent physics-based quantities and the regressor, which consists of the sensor measurements can be considered to be independent from the measurements at previous time instants. Furthermore, the case when the signs of the parameters are known and do not change can be easily reduced to the case when parameters are positive by re-defining the regressor components.

In this paper, we treat the set-membership identification problem of time-varying parameters in linear models and we account for both additive uncertainties in u_k and multiplicative uncertainties in ϕ_k. Under our assumptions, including boundedness of the time-varying parameters, the FSS is convex and can be computed by a recursive formula, which is formed by intersecting a prior estimate and a polyhedron. This polyhedron is defined by an information set, which consists of all the constraints on the feasible parameters obtained from the m measurements at the current time step.

In order to build corresponding AFSS for the FSS at each time step, two algorithms are developed in this paper. They are distinguished by their ways of processing the m measurements in y_k as well as the criterion of AFSS construction. These m measurements are segmented into l subsets ($l \le m$), each of which is referred to as a mini-batch in this paper. Within one iteration, the first algorithm processes a single measurement in y_k, which defines a convex cone constraint on the feasible parameters. Then, this algorithm computes candidate zonotopes that overbound the intersection between the prior estimate and the cone. Among all those candidates, the one with minimal estimated volume is selected and used as a prior estimate for the next iteration. After m iterations, all the measurements are processed and an AFSS for the FSS at the current time step is obtained. For handling time-varying parameters, the updated AFSS is propagated forward based on the rate of variations, providing a prior estimate of the AFSS for the next time step. This algorithm is referred to as CAZI which stands for Cone And Zonotope Intersection. In contrast to the first algorithm, the second algorithm processes multiple measurements in each iteration. This algorithm focuses on building a zonotope that overbounds the intersection between the prior estimate and a polyhedron, which is defined by the constraints associated with the current mini-batch of measurements. The number of measurements collected in each mini-batch is limited in order to reduce the computational complexity. An optimization problem subject to constraints prescribed by linear matrix inequalities (LMIs) is subsequently derived for computing the AFSS at each iteration. The P-radius (Le et al., 2013) of the resulting zonotope is minimized by solving this optimization problem. The propagations of the solutions from one iteration to another are as in the CAZI algorithm. Since the second algorithm emphasizes the way of building overbounds on Polyhedron And Zonotope Intersection, we refer to it as PAZI algorithm. Furthermore, PAZI algorithm is amenable to the analysis of the AFSS evolution over iterations. It is also found that the feasibility of the LMIs is closely related to signal richness and uncertainty level of the measurements. For illustrating the algorithms, an application to the engine condition monitoring problem with unknown health parameters is considered. Previous research addressing set-membership fault diagnosis may refer to Blesa, Puig, and Saludes (2011), Reppa and Tzes (2016), where only additive uncertainties are considered.

Our earlier conference paper (Wang, Kolmanovsky, & Sun, 2017) has attempted to solve the identification problem with single measurement at each time step (i.e. $y_k \in \mathbf{R}$) and has focused on the CAZI algorithm. In this paper, this problem is generalized to the multiple-measurement case, a new PAZI algorithm is introduced to handle this problem and the estimation performance is compared with the one of the CAZI algorithm.

The rest of the paper is organized as follows. Mathematical preliminaries are reviewed in Section 2. The problem formulation and the FSS properties are discussed in Section 3. The CAZI and PAZI algorithms are introduced in Section 4 and Section 5, respectively. In Section 6, the two algorithms are applied to an engine condition monitoring problem. An example of identifying time-varying parameters is reported in Section 7. The conclusions are presented in Section 8.

2. Preliminaries

Before we proceed with the detailed problem formulation, the following definitions are given since they are used throughout the paper.

Definition 1 (*Polyhedron*). A polyhedron is a convex set defined by intersecting a finite number of half spaces, $\mathcal{P} = \{\hat{\theta} \in \mathbf{R}^n : A\hat{\theta} \le b\}$,[1] where $A \in \mathbf{R}^{m\times n}$, $b \in \mathbf{R}^m$.

Definition 2 (*Polytope*). A polytope is a bounded polyhedron.

Definition 3 (*Minkowski Sum*). The Minkowski sum of two sets X and Y in the same linear space, denoted as $X \oplus Y$, is a set defined as $X \oplus Y = \{z : \exists x \in X, y \in Y \text{ such that } z = x + y\}$.

[1] The inequalities in this paper are element-by-element if not further defined.