Automatica 96 (2018) 134–140

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Asynchronous distributed localization in networks with communication delays and packet losses☆

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ARTICLE INFO

Article history:
Received 29 March 2017
Received in revised form 22 January 2018
Accepted 16 June 2018

Keywords:
Network localization
Asynchronous algorithm
Barycentric coordinate
Time-varying delay
Packet loss

ABSTRACT

This paper studies the problem of determining sensor locations in a large sensor network using only relative distance (range) measurement. Based on the barycentric coordinate representation, we propose a totally asynchronous distributed algorithm under DILOC framework due to independence of sensor update instants and unreliable networks with communication delays and packet losses. Through modeling the asynchronous algorithm as a linear difference equation with time-varying delays, we prove that the location estimates of sensors are globally convergent to the true coordinates if: (1) time interval between any two consecutive update instants is bounded from below and above, (2) communication delays and successive packet losses between sensors are finite. Simulation examples are provided to demonstrate the effectiveness of the theoretical result.

1. Introduction

Node self-localization is of essential significance for WSNs applied in GPS-denied areas, and is attracting more and more attention in recent years. However, how to get good performance such as global convergence and high accuracy is still a challenging problem in the design of distributed localization algorithms. For example, the second-order cone programming (SOCP) (Doherty, El Ghaoui, et al., 2001; Tseng & Paul, 2007) has a simpler structure and allows efficient distributed implementation, but it can just accurately position up to 80—90% of the sensors; the MDS-based distributed algorithm (Costa, Patwari, & Hero III, 2006) or gradient-based distributed localization algorithm (Todescato, Carron, Carli, Franchi, & Schenato, 2016) ensures only local convergence, which implies sensors have to be endowed with some GPS-like instrument for initialization. The distributed iterative localization (DILOC) algorithm proposed by Khan, Kar, and Moura (2009) has been proved to be globally convergent to the true coordinates if some node deployment restriction is satisfied. The main idea of DILOC is to express each sensor's location in a barycentric coordinate representation and convert the sensor localization problem into solving a pseudo linear system with all the nonlinearities hidden in the measured distances.

So far, most localization algorithms in the literature including DILOC are established under the assumption of ideal communication channel and time-invariant network topology. However, due to the complexity of the practical environment, limited communication range and noisy channels both interfere with the information transmission between sensors. The problem of the effect of communication delays and packet losses on the convergence and accuracy of localization algorithms has not been fully addressed. In D'Amato, Notaro, Mattei, and Tartaglione (2015), only the case of a constant delay is simulated with some numerical analysis. In Chagas and Waldmann (2015), the convergence of the localization algorithm considering the measurement delay is ensured only if additional information from GPS is gathered besides relative measurements. If relative positions between nodes are measurable, the localization problem can be handled as a consensus problem with a real or complex Laplacian, and the global convergence of localization algorithms can be established within a linear framework even for cases when measurement noise, asynchronous communication, and/or switching topology are involved in the localization process (Carron, Todescato, Carli, & Schenato, 2014; Lin, Han, Zheng, & Yu, 2017). However, if only range measurement is available, there is no reference to transform the localization problem to the consensus problem directly, to the best knowledge of the authors.

Besides, most localization algorithms in the literature are given in a fully synchronous manner, and only a few references have discussed asynchronous methods. By using a sequential greedy

☆ This work was supported by National Natural Science Foundation (NNSF) of China under Grant 61573105. The material in this paper was partially presented at the 56th IEEE Conference on Decision and Control, December 12–15, 2017, Melbourne, Australia. This paper was recommended for publication in revised form by Associate Editor Bert Tanner under the direction of Editor Christos G. Cassandras.
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https://doi.org/10.1016/j.automatica.2018.06.048
0005-1098/

optimization (SGO) method, partially asynchronous localization algorithm has been proposed in Shi, He, Chen, and Jiang (2010). In Srirangarajan, Tewfik, and Luo (2008), a totally asynchronous distributed localization algorithm has been proposed by using SOCP, but the convergence of the algorithm is shown numerically, not theoretically. In Todescato et al. (2016), an asynchronous gradient-based localization algorithm has been provided for networks with lossy communication. The algorithm is proved to be robust to packet losses and random delays with integration of GPS information for initialization and precise relative position measurements.

The main purpose of this paper is to design an asynchronous and distributed communication protocol for the barycentric-coordinate-based localization algorithm that uses only range measurement and ensures global convergence. Our work generalizes the DILOC algorithm in a totally asynchronous manner. The algorithm proposed in the paper is robust against any bounded time-varying communication delays and finite packet losses between neighboring nodes. By introducing some suitable transformation of system matrix and casting the iteration of the localization algorithm as a consensus problem, we theoretically prove that the algorithm is globally convergent to the true coordinates under these communication uncertainties. Some existed results on asynchronous consensus algorithms, such as extended delay graph and quotient graph (Cao, Morse, & Anderson, 2008), switching frequency estimate (Xiao & Wang, 2008), provide crucial tools for our convergence analysis.

Notations: Let $\mathbb{N}_+ = \{0, 1, 2, \ldots\}$, $\mathcal{I}_n = \{1, 2, \ldots, n\}$ and $\mathbf{0}$ be compatible dimensions of zeros. I_n denotes the identity matrix of order n. We say that $A \geq 0$ (A is nonnegative) if all its entries are nonnegative. A nonnegative matrix $A \in \mathbb{R}^{n\times n}$ with the property that all its row sums are $+1$ is said to be a stochastic matrix. For any nonnegative matrix $B = [b_{ij}] \in \mathbb{R}^{n\times n}$, let $\Lambda(B) = \{A = [a_{ij}] \mid a_{ij} \textit{ equals } b_{ij} \textit{ or } 0\}$. We let $\prod_{i=1}^{k} A_i = A_k A_{k-1} \cdots A_1$ denote the left product of matrices. Let $|\cdot|$ mean the cardinality of the set. $\lfloor a/b \rfloor$ means the maximum integer not greater than a/b.

2. Preliminaries

2.1. Graph theory

A digraph $G(\mathcal{V}, \mathcal{E})$ consists of a nonempty finite set $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ of nodes and a finite set $\mathcal{E} \subset \{(v_i, v_j) : v_i, v_j \in \mathcal{V}\}$ of arcs, i.e., ordered pairs of nodes. Here, we allow for self-loops, namely, such arcs as (v_i, v_i). For $(v_i, v_j) \in \mathcal{E}(G)$, v_i is called parent node, v_j is called child node, and v_i is also called a neighbor of v_j. The set of neighbors of v_i is defined by $\mathcal{N}_i = \{v_j : (v_j, v_i) \in \mathcal{E}\}$. A node v_i in G is said to be a neighbor of subset $\mathcal{U} \subseteq \mathcal{V}$ if v_i is a neighbor of at least one node in $\mathcal{U}$. A sub-digraph $G_s(\mathcal{V}_s, \mathcal{E}_s)$ of a digraph G is a digraph such that the node set $\mathcal{V}_s \subseteq \mathcal{V}$ and the arc set $\mathcal{E}_s \subseteq \mathcal{E}$. A path in G is a sequence $v_{i_1}, \ldots, v_{i_k}$ of nodes such that $(v_{i_j}, v_{i_{j+1}}) \in \mathcal{E}(G)$ for $j = 1, \ldots, k-1$.

For two digraphs $G_1 = (\mathcal{V}, \mathcal{E}_1)$ and $G_2 = (\mathcal{V}, \mathcal{E}_2)$, their union is denoted by $G_1 \cup G_2 := G(\mathcal{V}, \mathcal{E}_1 \cup \mathcal{E}_2)$; and their composition is denoted by $G_2 \circ G_1 := G(\mathcal{V}, \mathcal{E}_c)$, which is defined such that $(v_i, v_j) \in \mathcal{E}_c$ if for some vertex v_k, $(v_i, v_k) \in \mathcal{E}_1$ and $(v_k, v_j) \in \mathcal{E}_2$. We write $\mathcal{G}_{sl} = \{G = (\mathcal{V}, \mathcal{E}) : (v_i, v_i) \in \mathcal{E}, \forall v_i \in \mathcal{V}\}$ for the set of all directed graphs with one self-arc at each node. For any $G_1, G_2 \in \mathcal{G}_{sl}$, as $(v_i, v_i) \in \mathcal{E}(G_1), (v_j, v_j) \in \mathcal{E}(G_2)$ for all $v_i, v_j \in \mathcal{V}$, $(v_i, v_j) \in \mathcal{E}(G_2 \circ G_1)$ if $(v_i, v_j) \in \mathcal{E}(G_2)$ or $(v_i, v_j) \in \mathcal{E}(G_1)$. Thus, we have $G_1 \cup G_2 \subset G_2 \circ G_1$.

A node v_i of digraph G is said to be a root of G if for each other node v_j in G, there is a path from v_i to v_j. Digraph G is said to be strongly rooted at v_i, if v_i is a neighbor of each other node of G. In this paper, for digraph sequence $\{G_k, k \in \mathbb{N}_+\}$ induced by $\mathcal{V}$, we say that $\{G_k, k \in \mathbb{N}_+\}$ is uniformly composed rooted if there exists an integer sequence $0=\kappa_0 < \kappa_1 < \cdots < \kappa_{l+1} \cdots$, where $0 < H < \kappa_{l+1} - \kappa_l \leq K < \infty$, such that $G_{Cl} = G_{\kappa_{l+1}-1} \circ \cdots \circ G_{\kappa_l+1} \circ G_{\kappa_l}$ is rooted for all $l \in \mathbb{N}_+$.

A weighted digraph $G(S)$ is a digraph G plus a nonnegative stochastic matrix $S = [s_{ij}]_{n\times n}$ such that $(v_i, v_j) \in \mathcal{E}(G) \Leftrightarrow s_{ji} > 0$. Since the product of stochastic matrices is still a stochastic matrix, $G(S_h) \circ \cdots \circ G(S_2) \circ G(S_1) = G(S_h \cdots S_2 S_1)$ holds for any $h \in \mathbb{N}_+$.

The following notions are taken from Cao et al. (2008).

Definition 1. A rooted digraph $G(\mathcal{V}, \mathcal{E})$ is said to be a hierarchical digraph with hierarchy $\{v_1, v_2, \ldots, v_n\}$ if it is possible to relabel the nodes as $\{v_1, v_2, \ldots, v_n\}$ in such a way so that v_1 is a root of G with a self-arc and for $i > 1$, v_i has a neighbor v_j lower in the hierarchy, where by "v_j is lower than v_i" we mean $j < i$.

Definition 2. A digraph $\mathbb{G}(\bar{\mathcal{V}}, \bar{\mathcal{E}})$ induced by $\bar{\mathcal{V}} = \mathcal{V}_1 \cup \mathcal{V}_2 \cup \cdots \cup \mathcal{V}_n$, where $\mathcal{V}_i = \{v_{i1}, v_{i2}, \ldots, v_{im_i}\}(m_i \geq 1)$, is said to be an extended delay graph if for each $i \in \mathcal{I}_n$ the following statements hold
(1) there are self-arcs at each node in $\{v_{11}, v_{21}, \ldots, v_{n1}\}$,
(2) for each $j \in \{2, 3, \ldots, m_i\}$, node v_{ij} has only one neighbor which is exactly $v_{i(j-1)}$,
(3) every neighbor of $\mathcal{V}_i$ which is not in $\mathcal{V}_i$ is a neighbor of v_{i1},
(4) the subgraph of $\mathbb{G}$ induced by $\mathcal{V}_i$ has $\{v_{i1}, v_{i2}, \ldots, v_{im_i}\}$ as a hierarchy.

Definition 3. For an extended delay graph $\mathbb{G}$ induced by node set $\bar{\mathcal{V}} = \mathcal{V}_1 \cup \mathcal{V}_2 \cup \cdots \cup \mathcal{V}_n$, $\mathbb{Q}$ is said to be the *quotient graph* of $\mathbb{G}$ with node set $\mathcal{V} = \{1, 2, \ldots, n\}$ provided that there is an arc (i, j) in $\mathbb{Q}$ if and only if $\mathbb{G}$ has an arc from some node in $\mathcal{V}_i$ to some node in $\mathcal{V}_j$.

Consider a typical situation that the node sets given in Definition 2, $\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_n$ contain the same number of nodes, i.e., $m_1 = m_2 = \cdots = m_n := m$. Let us sort all the nodes of the extended delay graph $\mathbb{G}$ by order $v_{11}, v_{21}, \ldots, v_{n1}, v_{12}, v_{22}, \ldots, v_{n2}, \ldots, v_{1m}, v_{2m}, \ldots, v_{nm}$. Denote $P \in \mathbb{R}^{(n\times m)\times(n\times m)}$ the adjacency matrix of $\mathbb{G}$ in such an order, which can be partitioned as follows:

$$P = \begin{bmatrix} P_{11} & \cdots & P_{1m} \\ \cdots & \cdots & \cdots \\ P_{m1} & \cdots & P_{mm} \end{bmatrix},$$

where $P_{ij} \in \mathbb{R}^{n\times n}$, $i, j \in \{1, \ldots, m\}$. According to Definitions 2 and 3, we know that $\sum_{j=1}^{m} P_{1j}$ is an adjacency matrix of $\mathbb{G}$'s quotient graph $\mathbb{Q}$.

The following lemma is a restatement of Theorem 2 in Cao et al. (2008).

Lemma 1. *For stochastic matrices $S(i)$ corresponding to extended delay graphs $\mathbb{G}(i)(i \in \mathbb{N}_+)$ with the same nodes, if the sequence of quotient graphs of $\{\mathbb{G}(i), i \in \mathbb{N}_+\}$ is uniformly composed rooted with one self-arc at each node, as $t \to \infty$ the matrix product $\prod_{i=1}^{t} S(i)$ converges exponentially to a matrix of the form $\mathbf{1}c$, where c is a nonnegative constant row vector.*

2.2. Synchronous DILOC algorithm without communication delay

We state the DILOC algorithm (Khan et al., 2009) in 2-dimension Euclidean space $\mathbb{R}^2$. Consider a WSN with n nodes. Let $\kappa = \{1, 2, 3\}$ be the set of anchors whose locations are known and denoted by $p_a = \begin{bmatrix} p_1^T & p_2^T & p_3^T \end{bmatrix}^T \in \mathbb{R}^{3\times 2}$. Let $\Omega = \{4, \ldots, n\}$ be the set of sensors whose locations are to be determined and denoted by $p_s = \begin{bmatrix} p_4^T & \cdots & p_n^T \end{bmatrix}^T \in \mathbb{R}^{(n-3)\times 2}$. There are two deployment characteristics required by DILOC, which are described as the following assumption:

Assumption 1. (1) All sensors lie in the convex hull of the anchors; (2) each sensor, other than anchors, lies in the convex hull of three neighbors.