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Brief paper

# Price of anarchy in electric vehicle charging control games: When Nash equilibria achieve social welfare[☆]

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ABSTRACT

We consider the problem of optimal charging of plug-in electric vehicles (PEVs). We treat this problem as a multi-agent game, where vehicles/agents are heterogeneous since they are subject to possibly different constraints. Under the assumption that electricity price is affine in total demand, we show that, for any finite number of heterogeneous agents, the PEV charging control game admits a unique Nash equilibrium, which is the optimizer of an auxiliary minimization program. We are also able to quantify the asymptotic behaviour of the *price of anarchy* for this class of games. More precisely, we prove that if the parameters defining the constraints of each vehicle are drawn randomly from a given distribution, then, the value of the game converges almost surely to the optimum of the cooperative problem counterpart as the number of agents tends to infinity. In the case of a discrete probability distribution, we provide a systematic way to abstract agents in homogeneous groups and show that, as the number of agents tends to infinity, the value of the game tends to a deterministic quantity.



## 1. Introduction

Electric vehicles obtain some or all of their energy from the electricity grid, and are typically referred to as plug-in electric vehicles (PEVs). Their penetration is expected to increase significantly, since, not only they contribute to pollution reduction, but, by charging over low electricity price periods, they also serve as virtual dynamic storage, contributing to the stability of the electric grid (see Callaway & Hiskens, 2011; Denholm & Short, 2006; Li, Brocanelli, Zhang, & Wang, 2014; Rahman & Shrestha, 1993). In an electric vehicle charging control context two cases can be distinguished. The first case refers to a set-up where vehicles are social welfare maximizing entities and cooperate in view of minimizing the overall population cost. Under this setting, Deori, Margellos, and Prandini (2016), Deori, Margellos, and Prandini (2018) and Gan, Topcu, and Low (2013) propose iterative schemes that involve every vehicle solving a local minimization program, and show convergence to the social welfare optimum. In the second case vehicles act as selfish agents that seek to minimize their local cost, without being concerned with social welfare paradigms. This gives rise to multi-agent non-cooperative games, and the main concern is the computation of Nash equilibrium strategies. A complete theoretical analysis is provided in Huang, Caines, and Malhame (2007) and Lasry and Lions (2007) for stochastic continuous-time problems, but in the absence of constraints. The deterministic, discrete-time problem variant, was investigated in Ma, Callaway, and Hiskens (2013), and was further extended in Grammatico, Parise, Colombino, and Lygeros (2016) and Parise, Colombino, Grammatico, and Lygeros (2014) to account for the presence of constraints. However, for any finite number of agents, an approximate Nash equilibrium is computed, while the exact Nash one is reached only in the limiting case where the number of agents tends to infinity. The recent work of Paccagnan, Kamgarpour, and Lygeros (2016) overcomes this issue under the assumption that vehicles are aware of the way the total population consumption affects the price that drives their behaviour.

One challenge associated with the aforementioned stream of literature is that there is no common awareness on how the resulting Nash equilibrium solution is related to the associated social welfare optimum. In this paper we follow a pricing set-up similar to the seminal paper by Arrow and Debreu (1954), and account for constraint heterogeneity by assuming that the parameters defining the constraints of each vehicle are drawn randomly from a given

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distribution. We consider a multi-stage variant of the problem, however, we assume the price is an affine function of the total consumption. Under this set-up, our paper provides the following contributions:

(1) We quantify, to the best of our knowledge for the first time, the limiting value of the *price of anarchy* (Koutsoupias & Papadimitriou, 2016) for this class of games. The price of anarchy provides the means to quantify the efficiency of Nash equilibria, and is defined as the ratio between the worst-case value of the game achieved by a Nash equilibrium (in our setting there is a unique one) and the social optimum. We prove that as the number of agents tends to infinity this ratio tends to one for almost any choice of the random heterogeneity parameters (Theorem 2). This result extends (Ma et al., 2013) to the case of heterogeneous agents that are subject to constraints, without resorting to approximate Nash equilibria and primal–dual algorithms as in Li and Zhang (2016). As a byproduct we show that, for any finite number of possibly heterogeneous agents, the PEV charging control game admits a unique Nash equilibrium, which is the minimizer of an auxiliary minimization program (Proposition 4); see also Deori et al. (2017b). This is due to the fact that the underlying game is potential (Facchinei, Piccialli, & Sciandrone, 2011), however, our proof line is different and is based on fixed-point theoretic results. This result opens the road for the use of iterative algorithms for decentralized computation of Nash equilibria (Deori et al., 2018; Gan et al., 2013; Paccagnan et al., 2016).

(2) We provide the discrete time counterpart of the mean-field game theoretic approach in Huang et al. (2007), treating heterogeneity in a probabilistic manner, thus complementing the deterministic approaches of Grammatico et al. (2016), Li and Zhang (2016) and Paccagnan et al. (2016). In particular, we show that if the distribution of the random parameters that render agents' constraints heterogeneous is discrete, agents can be abstracted in homogeneous groups and, for almost any realization of the random heterogeneity parameters, as the number of agents tends to infinity, the value of the game tends to a deterministic quantity (Theorem 3).

It should be noted that our set-up exhibits similarities with multi-participant market investigations in Caramanis and Foster (2011), Caramanis, Goldis, Ruiz, and Rudkevich (2012), Caramanis, Ntakou, Hogan, Chakrabortty, and Schoene (2016) and Huang, Roozbehani, and Dahleh (2015). In particular, it is shown in Caramanis et al. (2012) that under current day-ahead operations participants have the incentive to self-dispatch, and the resulting social welfare market clearing prices are not practically viable. This is not in contrast with our results, since we show that Nash equilibria and social optima tend to coincide only in the limiting case of an infinite number of agents, and may differ for finite populations. Moreover, we consider a stylized architecture without including a distribution network model.

Section 2 introduces the non-cooperative PEV charging control game and its social welfare counterpart. Section 3 quantifies the price of anarchy for the limiting case of an infinite number of agents. In Section 4, we investigate the effect heterogeneity has in the value of the game, while Section 5 provides some directions for future work.

## 2. Electric vehicle charging control problem

### 2.1. Cooperative set-up

We first consider the case of $m$ PEVs that seek to determine their charging profile along some discrete time horizon $[0, h-1]$ of arbitrary length $h \in \mathbb{N}$ so as to minimize the total charging cost for the entire fleet. This corresponds to a cooperative set-up that is likely to occur when vehicles belong to the same managing entity. To this end, let $H = \{0, 1, \dots, h-1\}$ and $I = \{1, \dots, m\}$. Consider the following optimization program:

$$\min_{\{x^{it}\in\mathbb{R}\}_{t\in H, i\in I}} \sum_{t\in H} p^t \Big(\sum_{i\in I} x^{it} + x^{0t}\Big)^2 \tag{1}$$

$$\text{subject to: } \sum_{t\in H} x^{it} = \gamma^i, \text{ for all } i \in I, \tag{2}$$

$$x^{it} \in [\underline{x}^{it}, \overline{x}^{it}], \text{ for all } t\in H,\ i \in I, \tag{3}$$

where $x^{it} \in \mathbb{R}$ is the charging rate of vehicle $i$, $i \in I$, at time $t$, $t \in H$, and $p^t \geq 0$ is an electricity price coefficient at time $t$. For each $t \in H$, we denote by $x^{0t} \geq 0$ the non-PEV demand which, for a fixed number of PEVs $m$, is treated as constant and not as an optimization variable in the optimization programs below. Similarly to Ma et al. (2013) and Parise et al. (2014), for all $t \in H$, we assume that $\lim_{m\to\infty} x^{0t}/m = \hat{x}^{0t}$ is constant, allowing the non-PEV demand to grow linearly in the number of agents $m$ if $\hat{x}^{0t} \neq 0$.

The price of electricity is given by $p^t(\sum_{i\in I} x^{it} + x^{0t})$, and is assumed to depend linearly on the total PEV and non-PEV demand through $p^t$. Dependency of price on the PEV demand is affine due the presence of $x^{0t}$. Our choice for an affine price function is a simplification over (Arrow & Debreu, 1954; Gan et al., 2013; Ma et al., 2013) where convex monotone increasing functions are allowed, and is motivated by Grammatico et al. (2016), where an affine function is also employed, as well as by the numerical investigations of Gharesifard, Basar, and Dominguez-Garcia (2016) (in the corresponding theoretical analysis more general functions are allowed). The slope of this function encodes the inverse of the price elasticity of demand, and is motivated by the fact that marginal prices in lossless unconstrained energy systems are affine functions of the total production/demand (Caramanis et al., 2016). The objective function in (1) encodes the total electricity cost over $[0, h-1]$. Constraint (2) represents a prescribed charging level $\gamma^i \in \mathbb{R}$, $\gamma^i > 0$, to be reached by each vehicle $i$ at the end of the considered time horizon $H$, whereas (3) imposes minimum ($\underline{x}^{it} \in \mathbb{R}$, $\underline{x}^{it} \geq 0$) and maximum ($\overline{x}^{it} \in \mathbb{R}$, $\overline{x}^{it} < \infty$) limits, respectively, on $x^{it}$.

For all $i \in I$, let $x^i = [x^{i0}, \dots, x^{i(h-1)}]^\top \in \mathbb{R}^{|H|}$, where $|\cdot|$ denotes the cardinality of its argument. Let also $f : \mathbb{R}^{|H|} \times \mathbb{R}^{(m-1)|H|} \to \mathbb{R}$ be such that, for all $i \in I$, for any $(x^i, x^{-i}) \in \mathbb{R}^{m|H|}$,

$$f(x^i, x^{-i}) = \sum_{t\in H} x^{it} p^t \Big(\sum_{j\in I, j\neq i} x^{jt} + x^{it} + x^{0t}\Big), \tag{4}$$

where by $x^{-i} \in \mathbb{R}^{(m-1)|H|}$ we imply a vector including the decision variables of all vehicles except vehicle $i$ (recall that $x^{0t}$ is constant for any fixed $m$ and hence not included in these vectors). Moreover, for all $i \in I$, let

$$X^i = \Big\{x^i \in \mathbb{R}^{|H|} : \sum_{t\in H} x^{it} = \gamma^i \text{ and } x^{it} \in [\underline{x}^{it}, \overline{x}^{it}], \text{ for all } t \in H\Big\}, \tag{5}$$

denote the constraint set corresponding to vehicle $i$. Let $x = (x^1, \dots, x^m)$ and $X = X^1 \times \cdots \times X^m$, and consider $f_0 : \mathbb{R}^{m|H|} \to \mathbb{R}$ such that $f_0(x) = \sum_{t\in H} x^{0t} p^t(\sum_{j\in I} x^{jt} + x^{0t})$, which represents the cost of non-PEV demand. We can then rewrite (1)–(3) as

$$\mathcal{P}: \min_{\{x^i\in X^i\}_{i\in I}} f_0(x) + \sum_{i\in I} f(x^i, x^{-i}), \tag{6}$$

and refer to its optimal solution as social optimum. Note that local utility functions that depend only on the decision vector $x^i$ of each vehicle $i$, $i \in I$, and are possibly different per vehicle, can be incorporated in $\mathcal{P}$ by means of an epigraphic reformulation (see Deori et al., 2016).