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Brief paper

Robust discerning controls for the operating modes of linear switched systems subject to bounded unknown inputs ☆

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ABSTRACT

A linear switched system with bounded unknown inputs is considered in this paper. The study of the controlled distinguishability (or discernibility) property of the operating modes of the system is addressed. This property ensures the existence of a control input that generates different output signals of the modes regardless of the initial state vector and the unknown inputs. Such control inputs are called discerning controls. The robustness problem of the discerning controls with respect to the unknown inputs is analyzed, namely: under which conditions does a discerning control of the unknown-input-free modes remains a discerning control for the perturbed modes? To solve this problem, the existence of a quantifier measuring the size of the unknown inputs that discerning controls of the unknown-input-free modes have to be robust to in order to remain discerning controls for the perturbed modes is shown. In addition to this robustness result, an algorithm is proposed to design discerning controls for perturbed modes. Finally, from this input design procedure it is proven that when bounded unknown inputs are considered, there is an equivalence between controlled distinguishability of the unknown-input-free modes and that of the perturbed modes.

1. Introduction

Special attention has been paid over the last two decades to the identification of the active mode of switched systems from the input and output measurements (Domlan, Ragot, & Maquin, 2007; Halimi, Millrioux, & Daafouz, 2015; Ríos, Mincarelli, Efimov, Perruquetti, & Davila, 2015; Santis & Benedetto, 2017; Tanwani, Shim, & Liberzon, 2013; Vidal, 2008). This estimation problem is conditioned by the input–output distinguishability of the modes i.e. their property to have different input–output behaviors (Babaali & Pappas, 2004; Halimi et al., 2015; Lou & Si, 2009; Vidal, Chiuso, Soatto, & Sastry, 2002). Many concepts of distinguishability can be encountered in the literature (Babaali & Pappas, 2004; Baglietto, Battistelli, & Scardovi, 2007; Baglietto, Battistelli, & Tesi, 2014; Gómez-Gutiérrez, Ramírez-Trevino, Ruiz-León, & Gennaro, 2012; Lou & Si, 2009; Lou & Yang, 2014; Motchon, Pekpe, & Cassar, 2017; Motchon, Pekpe, Cassar, & Bièvre, 2016; Rosa & Silvestre, 2011). Among these, controlled distinguishability (Babaali & Pappas, 2004; Baglietto et al., 2007; Gómez-Gutiérrez et al., 2012) has attracted particular interests in the last few years. It ensures the existence of a control input generating different output signals for the modes regardless of their initial state vectors. This concept is introduced in Babaali and Pappas (2004) and this paper is mainly focused on the yes-or-no answer to the question of controlled distinguishability of the modes for unknown-input-free switched linear systems. In this context, it shows that the non-equality of the Markov parameters of the modes is a necessary and sufficient condition for the existence of a control input that makes the outputs of the modes distinguishable. The term "discerning controls" is used in this paper to refer to these control inputs. Moreover under the previous existence condition, Babaali and Pappas (2004) gives an input derivatives condition to characterize discerning controls. Determination of discerning control in unknown-input-free situation is also addressed for linear systems in Motchon et al. (2016) and for nonlinear control-affine systems in Motchon (2016) and Motchon et al. (2017), where approaches based on the explicit identification of the indistinguishability zone of the modes are proposed to tackle this problem.

In the present paper, we focus on controlled distinguishability problem for operating modes of a switched system subject to unknown inputs. Discerning controls of two modes of the system are inputs that generate different outputs of the two modes regardless

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of their initial state vectors and the unknown inputs of the system. We aim to analyze the following robustness and input design problems:

- Consider two operating modes of a switched system with unknown inputs and assume that in unknown-input-free situation, the considered modes are controlled distinguishable. Given a discerning control of the modes in the unknown-input-free case, under which conditions does this input remains a discerning control for the perturbed modes?
- How to design a discerning control of two controlled distinguishable modes in the presence of unknown inputs?

A quantifier which measures the size of the unknown inputs that discerning controls have to be robust to is introduced to tackle the robustness problem and an algorithm is proposed for solving the input design problem. In addition to these two contributions, we prove that controlled distinguishability of the unknown-input-free modes of switched systems with bounded unknown inputs is equivalent to that of the perturbed modes.

Controlled distinguishability of switched systems with unknown inputs has been also studied in Baglietto et al. (2007) and Gómez-Gutiérrez et al. (2012). In Baglietto et al. (2007), the characterization of the discerning controls provided in Babaali and Pappas (2004) is extended to the class of discrete-time switched systems with unknown bounded noises vectors. In Gómez-Gutiérrez et al. (2012), a geometric characterization of controlled distinguishability based on the notion of invariant space is given for switched continuous-time linear systems with disturbances.

The paper is organized as follows. The formal definitions of discerning controls and controlled distinguishability, and some general notation adopted for the modes are introduced in Section 2. Some preliminary results are also established in this section. The three contributions of the paper listed above are presented in Section 3 and are illustrated in Section 4 through a numerical example.

The following notations are adopted throughout the rest of the paper:

Symbols and notations	
0_s	Zero vector of $\mathbb{R}^s$
$0_{s\times r}$	Zero matrix of $\mathbb{R}^{s\times r}$
$\Vert\cdot\Vert_2$	Euclidean norm
$\langle\xi_1,\xi_2\rangle_M$	Weighted inner product of ξ_1 and ξ_2 associated to matrix M i.e. $\langle\xi_1,\xi_2\rangle_M=\xi_2^\top M\,\xi_1$
$\Vert\cdot\Vert_M$	Weighted norm associated to the inner product $\langle\cdot,\cdot\rangle_M$
$\lambda_{\min}(M)$	Minimum eigenvalue of matrix M
$\lambda_{\max}(M)$	Maximum eigenvalue of matrix M
$\mathbf{0}^{[s]}$	$\mathbf{0}^{[s]}:\mathbb{R}_+\to\mathbb{R}^s, t\mapsto 0_s$ is the zero valued function from $\mathbb{R}_+$ to $\mathbb{R}^s$.
$L^2([0;T],\mathbb{R}^s)$	Set of all measurable functions $\varphi:[0;T]\to\mathbb{R}^s$ such that $\int_0^T\Vert\varphi(\tau)\Vert_2^2\,d\tau<\infty$
$\langle\varphi,\psi\rangle_{L^2}$	$\langle\varphi,\psi\rangle_{L^2}=\int_0^T\varphi^\top(\tau)\,\psi(\tau)\,d\tau$ is the L^2-inner product of φ and ψ
$\Vert\varphi\Vert_{L^2}$	$\Vert\varphi\Vert_{L^2}=\left(\int_0^T\Vert\varphi(\tau)\Vert_2^2\,d\tau\right)^{1/2}$ is the L^2-norm of φ
$\mathscr{B}_{L^2}(\varphi,\rho)$	Closed ball of radius $\rho>0$ centered at $\varphi\in L^2([0;T],\mathbb{R}^s)$
$\mathbf{d}_{L^2}$	Metric induced by the norm $\Vert\cdot\Vert_{L^2}$. Given a set $\mathscr{S}\subset L^2([0;T],\mathbb{R}^s)$ and a function $\psi\in L^2([0;T],\mathbb{R}^s)$, $\mathbf{d}_{L^2}(\psi,\mathscr{S})=\inf_{\varphi\in\mathscr{S}}\Vert\varphi-\psi\Vert_{L^2}$ is the L^2-distance from ψ to $\mathscr{S}$.

2. Preliminaries

2.1. Discerning control of two modes

Let us consider the following operating modes S_p and S_q of a switched system with unknown switching signal: $i\in\{p,q\}$,

$$S_i\quad\begin{cases}\dot{x}_i(t)=A_i\,x_i(t)+B_i\,u(t)+E_i\,w(t)\\ y_i(t)=C_i\,x_i(t)+D_i\,u(t)+H_i\,w(t)\\ x_i(0)=x_i^o\end{cases}\tag{1}$$

where for every $t\in\mathbb{R}_+$, $x_i(t)\in\mathbb{R}^n$ and $y_i(t)\in\mathbb{R}^m$ represent respectively the state vector and the output vector of S_i, $u(t)\in\mathbb{R}^l$ denotes the known input vector of the switched system and $w(t)\in\mathbb{R}^d$ is the vector of unknown deterministic inputs acting on the switched system. The class of admissible control inputs and unknown inputs of the switched system are denoted by $\mathscr{U}$ and $\mathscr{W}$, respectively, and are assumed to satisfy the following hypotheses:

Assumption 1. The domain $\mathscr{W}$ is a closed, bounded and convex subset of $L^2\left([0;T],\mathbb{R}^d\right)$, and $\mathbf{0}^{[d]}\in\mathscr{W}$; $\mathscr{U}$ is the set of piecewise-continuous inputs defined on $\mathbb{R}_+$ with values in $\mathbb{R}^l$.

For clarity, we denote by $y_i\left(\cdot,x_i^o,u,w\right)$ the output of S_i generated by the initial condition x_i^o and the inputs u and w.

Definition 2 (*Discerning Control*)**.** An input $u\in\mathscr{U}$ is a $(\mathbb{R}^n,\mathscr{W})$-discerning control of modes S_p and S_q on $[0;T]$ if $\forall\left(x_p^o,x_q^o\right)\in\mathbb{R}^n\times\mathbb{R}^n$ and $\forall(w,\widetilde{w})\in\mathscr{W}\times\mathscr{W}$, $y_p\left(\cdot,x_p^o,u,w\right)\neq_{[0;T]}y_q\left(\cdot,x_q^o,u,\widetilde{w}\right)$.

Definition 3 (*Controlled Distinguishability*)**.** The modes S_p and S_q are said to be $(u,\mathbb{R}^n,\mathscr{W})$-controlled distinguishable on $[0;T]$ if u is a $(\mathbb{R}^n,\mathscr{W})$-discerning control of the modes on $[0;T]$. They are said to be $(\mathscr{U},\mathbb{R}^n,\mathscr{W})$-controlled distinguishable on $[0;T]$ if there exists at least an input $u\in\mathscr{U}$ which is a $(u,\mathbb{R}^n,\mathscr{W})$-discerning control of the modes on $[0;T]$.

Remark 4. For simplicity purpose, in the presentation, we study the problem of distinguishability on $[0;T]$. The results can be generalized easily to intervals of form $[t_0;t_0+T]$ with $t_0\geq 0$. The controlled distinguishability of the operating modes w.r.t. a given input u guarantees that when no switching occurs during the period of time $[0;T]$, the active mode can be identified using u and the output of the system generated by this input. Then performing controlled distinguishability test on $[0;T]$ gives the capacity of the input u to detect the active mode on this interval. In application, for switched systems with minimum dwell time, the final time of observation T can represent this value.

2.2. Augmented system of two modes

A main tool for our subsequent analysis is the augmented system S_{pq} associated to modes S_p and S_q:

$$S_{pq}\quad\begin{cases}\dot{x}_{pq}(t)=A_{pq}\,x_{pq}(t)+B_{pq}\,u(t)+E_{pq}\,\overline{\mathrm{w}}(t)\\ y_{pq}(t)=C_{pq}\,x_{pq}(t)+D_{pq}\,u(t)+H_{pq}\,\overline{\mathrm{w}}(t)\\ x_{pq}(0)=x_{pq}^o\end{cases}$$

where $x_{pq}(t)\in\mathbb{R}^{2n}$, $\overline{\mathrm{w}}(t)\in\mathbb{R}^{2d}$ and $y_{pq}(t)\in\mathbb{R}^m$ are respectively the state vector, the unknown input vector and the output vector of the augmented system. The matrices $A_{pq}\in\mathbb{R}^{2n\times 2n}$, $B_{pq}\in\mathbb{R}^{2n\times l}$, $C_{pq}\in\mathbb{R}^{m\times 2n}$, $D_{pq}\in\mathbb{R}^{m\times l}$, $E_{pq}\in\mathbb{R}^{2n\times 2d}$ and $H_{pq}\in\mathbb{R}^{m\times 2d}$ are defined by:

$$A_{pq}=\begin{bmatrix}A_p & 0_{n\times n}\\ 0_{n\times n} & A_q\end{bmatrix},\ B_{pq}=\begin{bmatrix}B_p\\ B_q\end{bmatrix},\ E_{pq}=\begin{bmatrix}E_p & 0_{n\times d}\\ 0_{n\times d} & E_q\end{bmatrix}$$

and

$$C_{pq}=\begin{bmatrix}C_p & -C_q\end{bmatrix},\ D_{pq}=D_p-D_q,\ H_{pq}=\begin{bmatrix}H_p & -H_q\end{bmatrix}.$$

The following notation is used for the augmented system:

Notation 1. *We denote by* $\mathbf{A}_{pq}$, $\mathbf{C}_{pq}$ *and* $\mathbf{Q}_{pq}$ *the matrices defined by*

$$\left(\mathbf{A}_{pq},\mathbf{C}_{pq},\mathbf{Q}_{pq}\right)=\begin{cases}\left(\widetilde{A}_{pq},\widetilde{C}_{pq},\widetilde{Q}_{pq}\right) & \text{if}\quad \mathbf{r}<2n\\ \left(A_{pq},C_{pq},I_{2n}\right) & \text{if}\quad \mathbf{r}=2n\end{cases}$$