Automatica 96 (2018) 234–239

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

A necessary and sufficient condition for local asymptotic stability of a class of nonlinear systems in the critical case[☆]

Jiandong Zhu[a,b,*], Chunjiang Qian[b]

[a] Institute of Mathematics, School of Mathematical Sciences, Nanjing Normal University, Nanjing, 210023, PR China
[b] Department of Electrical and Computer Engineering, University of Texas at San Antonio, San Antonio, TX 78249, USA

ARTICLE INFO

Article history:
Received 21 May 2017
Received in revised form 7 May 2018
Accepted 7 June 2018

Keywords:
Local asymptotic stability
Nonlinear system
Chain of power integrators
Lyapunov/Chetaev function

ABSTRACT

By the theory of linear differential equations, a system described by a chain of integrators with a linear feedback is globally asymptotically stable if and only if the characteristic polynomial is Hurwitz, which implies that all the coefficients in the linear feedback equation are negative. However, negative coefficients may not guarantee the local asymptotic stability of the linear system. In this paper, we reveal that, by monotonizing the powers of the integrators, the strict negativity of the feedback coefficients is not only necessary but also sufficient for the local asymptotic stability of the system. A dual result is also obtained for the dual power integrator systems.

© 2018 Elsevier Ltd. All rights reserved.

1. Introduction

In this paper, we are interested in stability analysis of a class of nonlinear systems described by

$$\begin{aligned} \dot{x}_i &= x_{i+1}^{p_i}, \quad i=1,2,\ldots,n-1, \\ \dot{x}_n &= (-k_1x_1-k_2x_2-\cdots-k_nx_n)^{p_n}, \end{aligned} \tag{1}$$

where $x=(x_1,x_2,\ldots,x_n)\in\Re^n$ is the state vector, $p_i\in\Re$ is a ratio of positive odd integers not less than one, and $k_i\in\Re$ is a constant, for $i=1,2,\ldots,n$. System (1) is a special case of the so-called p-normal form to which some nonlinear control systems can be equivalently transformed under certain conditions (Cheng & Lin, 2003; Respondek, 2003). When $p_i=1$, $i=1,\ldots,n$, system (1) becomes a linear system whose stability analysis has been well-studied. In fact, it is known that the linear system $\dot{x}=Ax$ is globally asymptotically stable if and only if the characteristic polynomial $\det(\lambda I-A)$ is Hurwitz, i.e. all the poles of the system have only negative real parts. For example, the following linear system described by a chain of integrators with a linear feedback

$$\begin{cases} \dot{x}_1 = x_2, \quad \dot{x}_2 = x_3, \\ \dot{x}_3 = -k_1x_1-k_2x_2-k_3x_3 \end{cases} \tag{2}$$

is globally asymptotically stable if and only if the polynomial $p(\lambda)=\lambda^3+k_3\lambda^2+k_2\lambda+k_1$ is Hurwitz, that is, the real part of every root of $p(\lambda)$ is negative. Criteria for being a Hurwitz polynomial were proposed on the selection of k_i's by Routh and Hurwitz independently (Bennett, 1979; Hurwitz, 1895; Routh, 1877).

For nonlinear systems, Lyapunov's first method (Lyapunov, 1892), also called linearization method, is one of the common methods to study stability. It shows that if the linearized system is globally asymptotically stable, i.e., every pole has a negative real part, the nonlinear system is locally asymptotically stable. Moreover, if the linearized system has a pole with a strictly positive real part, the nonlinear system is unstable. It is worth noting that, if the linearized system has no poles with strictly positive real parts but has poles lying on the imaginary axis, no conclusion can be drawn about the original nonlinear system. This kind of nonlinear systems are known as critical cases, and system (1) falls in this category whenever there exists $p_i>1$.

Reduction Principle based on Center Manifold Theorem (Isidori, 1995; Pliss, 1964) is an important tool for the critical case, but it requires that the linearized system has a pole with a negative real part. In this case, the stability problem of the original nonlinear system can be reduced to that of a lower-dimensional system. For example, the stability analysis of (1) with $p_n=1$ and $k_n>0$ can be reduced to that of an $(n-1)$-dimensional systems. But the stability problem of the lower-dimensional system is still not

☆ This work is supported in part by National Natural Science Foundation (NNSF) of China under Grants 61628302, 61673012 and a project funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD). The material in this paper was partially presented at the 11th Asian Control Conference, December 17–20, 2017, Gold Coast, Australia. This paper was recommended for publication in revised form by Associate Editor Romain Postoyan under the direction of Editor Daniel Liberzon.

* Corresponding author at: Institute of Mathematics, School of Mathematical Sciences, Nanjing Normal University, Nanjing, 210023, PR China.
E-mail addresses: zhujiandong@njnu.edu.cn (J. Zhu), Chunjiang.Qian@utsa.edu (C. Qian).

https://doi.org/10.1016/j.automatica.2018.06.052
0005-1098/© 2018 Elsevier Ltd. All rights reserved.

easily solvable. In addition, when $p_i > 1$ for all i, the Reduction Principle fails. In this case, the most effective method is the Lyapunov's second or direct method, as well as its extension known as Barbashin–Krasovskii–LaSalle Principle (Barbashin & Krasovskii, 1952; LaSalle, 1960; Arsie & Ebenbauer, 2010).

For Lyapunov's second method, the crux of stability analysis is seeking a suitable Lyapunov function. For nonlinear control systems in the p-normal form, the technique of adding a power integrator (AAPI) was proposed in Lin and Qian (2000) to construct Lyapunov functions and design high-gain nonlinear feedback controllers. AAPI technique has been widely applied to many control systems (Ding & Zheng, 2016; Qian & Lin, 2001; Sun, Li, & Yang, 2016; Sun, Xue, & Zhang, 2015; Zhai & Fei, 2011). However, due to its domination nature, the nonlinear controllers designed by the AAPI technique lead to very large control parameters which increase significantly along with the dimension of the controlled system and consequently cause implementation issues in practice. Compared with nonlinear feedback, linear feedback has simple controller structure that can be easily implemented by means of usual engineering devices. Using a linear feedback to stabilize a nonlinear system has been investigated in many contributions such as (Fu & Abed, 1993; Tsinias, 1991). For a chain of power integrators with linear feedback such as the simple example

$$\begin{cases}\dot{x}_1 = x_2^5, \ \ \dot{x}_2 = x_3^3, \\ \dot{x}_3 = -k_1x_1 - k_2x_2 - k_3x_3,\end{cases} \tag{3}$$

a natural question is what is the sufficient condition on the feedback gains for the local asymptotic stability of (3). Another interesting question is if it is necessary for the local asymptotic stability of (3) to require the polynomial $p(\lambda) = \lambda^3 + k_3\lambda^2 + k_2\lambda + k_1$ to be Hurwitz.

In this paper, based on the idea of homogeneity with monotone degrees (Polendo & Qian, 2008; Su, Qian, & Shen, 2017), we will tackle these problems. We shall prove that any strictly positive parameters k_i's will be sufficient to guarantee local asymptotic stability of (3). In other words, even though the coefficients do not define a Hurwitz polynomial, (3) is still locally asymptotically stable as long as the k_i's are strictly positive. For instance, when $k_1 = 2, k_2 = k_3 = 1$, even though the linear system (2) is unstable since $\lambda^3 + \lambda^2 + \lambda + 2 = 0$ has a pair of unstable roots, (3) is however locally asymptotically stable. A surprising conclusion is that, as $k_i \neq 0$ $(i = 1, 2, 3)$, system (3) is locally asymptotically stable if and only if $k_i > 0$ $(i = 1, 2, 3)$. The same conclusion can be drawn for the general nonlinear systems (1) if the powers, i.e., p_i's, are strictly decreasing. Moreover, a dual nonlinear system of the chain of power integrators is proposed and the dual result on local asymptotic stability is obtained.

2. Preliminaries

In this section, we present some important results on nonlinear systems, which will be used in this paper.

Consider the autonomous system $\dot{x} = f(x)$, where $f : D \to \Re^n$ is a locally Lipschitz map from a domain $D \subset \Re^n$. Assume that $x = 0$ is an equilibrium, that is, $f(0) = 0$.

Theorem 2.1 (*Lyapunov Stability Theorem (Khalil, 2002; Loria & Panteley, 2017; Lyapunov, 1892)*). *If there exists a continuously differentiable locally positive (negative) definite function $V : D \to \Re$ so that $\dot{V}(x) := \frac{\partial V}{\partial x}f(x)$ is locally negative (positive) definite, the equilibrium $x = 0$ is locally asymptotically stable.*

Theorem 2.2 (*Chetaev Instability Theorem (Chetaev, 1961)*).

If there exists a continuously differentiable function $V : D \to \Re$ such that (i) *the origin is a boundary point of the set $G = \{x \in \Re^n \mid V(x) > 0\}$;* (ii) *there exists a neighborhood U of the $x = 0$ such that $\dot{V}(x) > 0 \ \forall x \in U \cap G$, then $x = 0$ is an unstable equilibrium point of the system.*

Lemma 2.1 (*Jensen's inequality (Hardy, Littlewood, & Pólya, 1934)*). *For $p \geq 1$ and $x_i \in \Re$, $i = 1, \dots, n$, the following holds:*

$$|x_1 + x_2 + \cdots + x_n|^p \leq n^{p-1}(|x_1|^p + |x_2|^p + \cdots + |x_n|^p).$$

Lemma 2.2 (*Wang, Qian, & Sun, 2017*). *For $p \geq 1$ which is a ratio of positive odd integers, the following holds:*

$$x(x + a)^p \geq 2^{1-p}x^{p+1} + xa^p, \ \forall x, \ a \in \Re.$$

Lemma 2.3 (*Polendo & Qian, 2007*). *Let $c > 0$ and $d > 0$ be constants. Given any number $\gamma > 0$, the following inequality holds:*

$$|x|^c|y|^d \leq \frac{c}{c+d}\gamma|x|^{c+d} + \frac{d}{c+d}\gamma^{-\frac{c}{d}}|y|^{c+d}, \forall x, \ y \in \Re.$$

Definition 2.1 (*Polendo & Qian, 2008*). A continuous vector field $f : \mathrm{R}^n \to \mathrm{R}^n$ with $f = [f_1, \dots, f_n]^{\mathrm{T}}$ is said to satisfy homogeneity with monotone degrees (HMD), if we can find strictly positive real numbers $(r_1, \dots, r_n)$ and real numbers $\tau_1 \geq \tau_2 \geq \cdots \geq \tau_n$ such that

$$f_i(\epsilon^{r_1}x_1, \dots, \epsilon^{r_n}x_n) = \epsilon^{r_i+\tau_i}f_i(x) \tag{4}$$

for all $x \in \Re^n$, $\epsilon > 0$ and $i = 1, 2, \dots, n$. The constants r_i's and τ_i's are called homogeneous weights and degrees, respectively.

3. Local asymptotic stability of power integrators systems

3.1. Main result

In this section, we consider a chain of power integrators described by (1) with strictly decreasing powers p_i's.

Theorem 3.1. *For nonlinear system (1), assume that $k_i \neq 0$ for all $i = 1, 2, \dots, n$ and p_i's are ratios of positive odd integers satisfying $p_1 > p_2 > \cdots > p_n \geq 1$. Then the equilibrium $x = 0$ is locally asymptotically stable if and only if $k_i > 0$ for all $i = 1, 2, \dots, n$.*

Proof. Construct a linear transformation

$$e_i = k_1x_1 + k_2x_2 + \cdots + k_ix_i, \ \ i = 1, 2, \dots, n, \tag{5}$$

whose inverse mapping is

$$x_1 = k_1^{-1}e_1, \ x_i = k_i^{-1}(e_i - e_{i-1}), \ \ i = 2, \dots, n. \tag{6}$$

Then system (1) is equivalently transformed into

$$\begin{aligned}\dot{e}_1 &= \frac{k_1}{k_2^{p_1}}(e_2-e_1)^{p_1}, \\ \dot{e}_i &= \frac{k_i}{k_{i+1}^{p_i}}(e_{i+1}-e_i)^{p_i} + \sum_{j=1}^{i-1}\frac{k_j}{k_{j+1}^{p_j}}(e_{j+1}-e_j)^{p_j}, \\ &\qquad\qquad i = 2, 3, \dots, n-1, \\ \dot{e}_n &= -k_ne_n^{p_n} + \sum_{j=1}^{n-1}\frac{k_j}{k_{j+1}^{p_j}}(e_{j+1}-e_j)^{p_j}.\end{aligned} \tag{7}$$

Construct the following Lyapunov/Chetaev function:

$$V(e) = \frac{1}{2}\sum_{i=1}^{n}l_ie_i^2, \tag{8}$$

where

$$l_i = -k_{i+1}^{p_i}k_i^{-1}, \ i = 1, 2, \dots, n-1, \ l_n = -k_n^{-1}. \tag{9}$$

Download English Version:

https://daneshyari.com/en/article/7108248

Download Persian Version:

https://daneshyari.com/article/7108248

Daneshyari.com