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Brief paper

Unified iterative learning control for flexible structures with input constraints☆

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ABSTRACT

This paper proposes a unified framework of iterative learning control for typical flexible structures under spatiotemporally varying disturbances. Input constraints and the external disturbances are smoothly tackled through hyperbolic tangent functions. Boundary iterative learning control (BILC) laws are proposed to guarantee the learning convergence. The closed-loop systems can converge to zero along the iteration axis on the basis of time-weighted Lyapunov–Krasovskii-like composite energy functions (CEF). Simulations are implemented to illustrate the effectiveness of the proposed BILC schemes.

1. Introduction

Flexible structures are widely used in robots, spacecrafts, marine risers, etc., for their light weight, low energy consumption and flexibility (Cai & Krstic, 2016; Do, 2017; Jin & Guo, 2015; Smyshlyaev, Guo, & Krstic, 2009; Wu & Wang, 2014). Based on extended Hamilton's principle, flexible structures are modeled as distributed parameter systems. Different from ordinary differential equation (ODE) systems (Ge, Hang, Lee, & Zhang, 2001; Krstic & Smyshlyaev, 2008; Li, & Yao, 2016; Liu & Tong, 2017; Yang & Liu, 2018), partial differential equation (PDE) systems are much more complex not only for two or more variables involved but also for the coupling in the variables (Guo & Jin, 2015a, b; Karafyllis & Krstic, 2018; Luo, Huang, Wu, & Yang, 2015; Wang, Liu, Ren, & Chen, 2015; Wang, Zhang, Wei, Zheng, & Li, 2017; Wu, Wang, & Guo, 2016; Wu & Zhu, 2017). In addition, external disturbances and nonlinear inputs are frequently encountered in engineering (Li, Chen, Fu, & Sun, 2016) and should be taken into consideration for a more accurate model (Fan, 2014; Zhang & Hou, 2015). Disturbances are rejected frequently by using symbolic functions (He, Meng, Huang, & Li, 2018; He, He, Shi, & Sun, 2017). Input saturation can be tackled by designing auxiliary systems (Sun, Huo, & Jiao, 2017; Zhou, Wang, Wu, Li, & Du, 2017). In Wen, Zhou, Liu, and Su (2011), input saturation, composed of a hyperbolic tangent function and a bounded term, was addressed through Nussbaum functions and adaptive laws.

Until now, some works have proposed iterative learning control (ILC) for distributed parameter systems (He et al., 2018; Huang & Xu, 2011; Huang, Xu, Li, Xu, & Yu, 2013; Meng & He, 2017; Qu, 2002; Zhao & Rahn, 2008). In Huang and Xu (2011) and Huang et al. (2013), ILC was designed for PDE systems under distributed disturbances, which were assumed to be Lipschitz continuous. In Qu (2002) and Zhao and Rahn (2008), without external disturbances and nonlinear inputs considered, a flexible moving string system was addressed by ILC. In order to tackle some complex issues, including output constraints, external disturbances, etc., boundary ILC (BILC) is designed by combining a traditional ILC law and feedback terms of such issues, at the basis of composite energy functions (CEFs) (Xu, Tan, & Lee, 2004). In He et al. (2018) and Meng and He (2017), symbolic functions were adopted in BILC laws for disturbance rejection and input saturation. However, this paper proposes a continuously differentiable BILC law for flexible structures.

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The remainder of this paper is organized as follows. In Section 2, system models are presented along the iteration axis. By proposing the unified BILC law in Section 3, learning convergence is proved in Section 4. Simulation examples are provided in Section 5. Section 6 summarizes the main results of this paper.

2. Problem formulation

This paper considers a flexible string, an Euler–Bernoulli beam and a Timoshenko beam with external disturbances and input constraint. In the above systems, rotation and transverse vibration are encountered, which are two basic motions for one-dimension flexible structures. Three common deformations, including shear deformation, torsion and bending, are all considered in the one-dimension flexible structures. In the presence of external disturbances and nonlinear inputs, the chosen flexible structures are common in engineering. Let $f_{sj}(x,t)$, $f_{ej}(x,t)$ and $f_{tj}(x,t)$ represent spatiotemporally varying disturbances. $d_{usj}(t)$, $d_{uej}(t)$, $d_{utj}(t)$ and $d_{\tau tj}(t)$ denote time-varying disturbances. The following assumption is made for the considered disturbances:

Assumption 1. External disturbances are assumed to be bounded and some known positive constants can be found, satisfying $|f_{sj}(x,t)| \le \bar{f}_s$, $|f_{ej}(x,t)| \le \bar{f}_e$, $|f_{tj}(x,t)| \le \bar{f}_t$, $|d_{usj}(t)| \le \bar{d}_{us}$, $|d_{uej}(t)| \le \bar{d}_{ue}$, $|d_{utj}(t)| \le \bar{d}_{ut}$ and $|d_{\tau tj}(t)| \le \bar{d}_{\tau t}$ for $\forall t \in [0, T_0]$ and $j \in \mathbb{N}$.

In the following tabular, E-beam and T-beam represent Euler–Bernoulli beam and Timoshenko beam, respectively.

Nomenclature	
ρ_s, ρ_e, ρ_t	Unit mass per unit length of the string, E-beam or T-beam
L_s, L_e, L_t	Length of the string, E-beam or T-beam
M_s, M_e, M_t	Mass of tip payload attached at the bottom of the string, E-beam or T-beam
T_s, T_e	Tension of the string or E-beam
EI_e, EI_t	Bending stiffness of the E-beam or T-beam
I_{pt}	Uniform mass moment of inertia of the cross section of T-beam
K_t	$k_t A_t G_t$
k_t	A positive constant depending on the shape of the cross section
A_t	Cross-sectional area
G_t	Modulus of elasticity in shear
J_t	Inertia of the tip payload attached at the bottom of T-beam
w_{sj}, w_{ej}, w_{tj}	Lateral deflections of the string, E-beam or T-beam
$\dot{w}_{sj}$, $\dot{w}_{ej}$, $\dot{w}_{tj}$	Velocity of the string, E-beam or T-beam
$\ddot{w}_{sj}$, $\ddot{w}_{ej}$, $\ddot{w}_{tj}$	Acceleration of the string, E-beam or T-beam
ϕ_{tj}	Angle displacement of the T-beam
$\dot{\phi}_{tj}$	Angle velocity of the T-beam
$\ddot{\phi}_{tj}$	Angle acceleration of the T-beam

As shown in He and Ge (2012) and He et al. (2018), three flexible structures can be described as

$$\begin{cases} \rho_s \ddot{w}_{sj}(x,t) &= f_{sj}(x,t) + T_s w''_{sj}(x,t), \\ w_{sj}(0,t) &= 0, \\ M_s \ddot{w}_{sj}(L_s,t) &= u^* \tanh(\frac{u_{0j}(t)}{u^*}) + d_{usj}(t) \\ & \quad -T_s w'_{sj}(L_s,t), \end{cases} \tag{1}$$

$$\begin{cases} \rho_e \ddot{w}_{ej}(x,t) &= f_{ej}(x,t) - EI_e w''''_{ej}(x,t) \\ & \quad +T_e w''_{ej}(x,t), \\ w_{ej}(0,t) &= 0, \\ w'_{ej}(0,t) &= 0, \\ w''_{ej}(L_e,t) &= 0, \\ M_e \ddot{w}_{ej}(L_e,t) &= u^* \tanh(\frac{u_{0j}(t)}{u^*}) + EI_e w'''_{ej}(L_e,t) \\ & \quad -T_e w'_{ej}(L_e,t) + d_{uej}(t), \end{cases} \tag{2}$$

$$\begin{cases} I_{\rho t} \ddot{\phi}_{tj}(x,t) &= EI_t \phi''_{tj}(x.t) - K_t[\phi_{tj}(x,t) - w'_{tj}(x,t)], \\ \rho_t \ddot{w}_{tj}(x,t) &= f_{tj}(x,t) - K_t[\phi'_{tj}(x,t) - w''_{tj}(x,t)], \\ w_{tj}(0,t) &= 0, \\ \phi_{tj}(0,t) &= 0, \\ M_t \ddot{w}_{tj}(L_t,t) &= u^* \tanh(\frac{u_{0j}(t)}{u^*}) + d_{utj}(t) \\ & \quad +K_t[\phi_{tj}(L_t,t) - w'_{tj}(L_t,t)], \\ J_t \ddot{\phi}_{tj}(L_t,t) &= \tau^* \tanh(\frac{\tau_{0j}(t)}{\tau^*}) - EI_t \phi'_{tj}(L_t,t) \\ & \quad +d_{\tau tj}(t), \end{cases} \tag{3}$$

where $u^* > 0$, $\tau^* > 0$ and $t \in [0, T_0]$.

Assumption 2 (Xu & Yan, 2004). In this paper, alignment conditions are assumed as follows:

(1) For the flexible string system, $w_{sj}(x,0) = w_{s(j-1)}(x,T_0)$ and $\dot{w}_{sj}(x,0) = \dot{w}_{s(j-1)}(x,T_0)$;

(2) For the Euler–Bernoulli beam system, $w_{ej}(x,0) = w_{e(j-1)}(x,T_0)$ and $\dot{w}_{ej}(x,0) = \dot{w}_{e(j-1)}(x,T_0)$;

(3) For the Timoshenko beam system, $w_{tj}(x,0) = w_{t(j-1)}(x,T_0)$, $\dot{w}_{tj}(x,0) = \dot{w}_{t(j-1)}(x,T_0)$, $\phi_{tj}(x,0) = \phi_{t(j-1)}(x,T_0)$ and $\dot{\phi}_{tj}(x,0) = \dot{\phi}_{t(j-1)}(x,T_0)$;

3. Control design

Comparing with the traditional ILC $u_{0j}(t) = u_{0(j-1)}(t) - F_{wj}(t)$ (Meng, Jia, Du, & Yu, 2012; Xu & Tan, 2003), the form is expanded in this paper: (I) the learning term $u_{0(j-1)}(t)$ is updated as $u_{1(j-1)}(t)$; (II) the feedback term $F_{wj}(t)$ is changed from state feedbacks to feedbacks of system states, external disturbances, nonlinear inputs, etc. Therefore, the BILC law in nature is an extended ILC law and is mostly model-free. In order to present the model-free merit of the BILC methodology, a unified but continuously differentiable BILC scheme is designed for three typical flexible structures.

The unified BILC law is expressed as

$$\begin{cases} u_{0j}(t) &= u_{1j}(t) - \nu_1 \kappa_f \tanh(\nu_5 \kappa_{wj}(t)) \\ & \quad -(\nu_2 \kappa_d + \nu_3 \kappa_u) \tanh(\nu_4 \kappa_{wj}(t)), \\ u_{1j}(t) &= u^* \tanh(\frac{u_{1(j-1)}(t)}{u^*}) - \gamma \kappa_{wj}(t), \end{cases} \tag{4}$$

where $u_{1(-1)}(t) = 0$, $\kappa_u = u_0^*(1 - \tanh(1)) > 0$ and ν_1-ν_5 and γ are positive constants. $\kappa_{wj}(t)$ denotes the feedback terms of system states. $\kappa_f > 0$ represents the upper bound of distributed disturbances. $\kappa_d > 0$ is the upper bound of boundary disturbances.

Property 1. *For the proposed BILC law (4), the following inequality is obtained (Xu et al., 2004):*

$$\begin{aligned} & |u_{0j}(t) - u^* \mathrm{sat}(\frac{u_{0j}(t)}{u^*}, 1)| \\ & \le (\gamma + \nu_1 \kappa_f \nu_5 + (\nu_2 \kappa_d + \nu_3 \kappa_u)\nu_4)|\kappa_{wj}(t)| \end{aligned} \tag{5}$$

where the saturated function is defined as

$$\mathrm{sat}(\frac{u_{0j}(t)}{u^*}, 1) = \begin{cases} \mathrm{sgn}(u_{0j}(t)), & \text{if } |u_{0j}(t)| \ge u^*, \\ \frac{u_{0j}(t)}{u^*}, & \text{if } |u_{0j}(t)| < u^*, \end{cases} \tag{6}$$