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Distributed sampled-data control of Kuramoto–Sivashinsky equation☆

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ABSTRACT

The paper is devoted to distributed sampled-data control of nonlinear PDE system governed by 1-D Kuramoto–Sivashinsky equation. It is assumed that N sensors provide sampled in time spatially distributed (either point or averaged) measurements of the state over N sampling spatial intervals. Locally stabilizing sampled-data controllers are designed that are applied through distributed in space shape functions and zero-order hold devices. Given upper bounds on the sampling intervals in time and in space, sufficient conditions ensuring regional exponential stability of the closed-loop system are established in terms of Linear Matrix Inequalities (LMIs) by using the time-delay approach to sampled-data control and Lyapunov–Krasovskii method. As it happened in the case of diffusion equation, the descriptor method appeared to be an efficient tool for the stability analysis of the sampled-data Kuramoto–Sivashinsky equation. An estimate on the domain of attraction is also given. A numerical example demonstrates the efficiency of the results.

1. Introduction

Kuramoto–Sivashinsky equation (KSE) describes a variety of physical and chemical phenomena including magnetized plasmas, flame front propagation, viscous flow problems and chemical reaction–diffusion processes (see e.g. Kuramoto & Tsuzuki, 1975; Sivashinsky, 1977; Lunasin & Titi, 2017). Boundary control of 1-D KSE was studied in Coron and Lü (2015) and Liu and Krstic (2001). The local rapid stabilization problem for a controlled KSE on a bounded interval was considered in Coron and Lü (2015). In Liu and Krstic (2001), a Neumann feedback law was designed to guarantee L^2-global exponential stability and H^2-global asymptotic stability for small values of the anti-diffusion parameter.

Distributed control of KSE was studied in Armaou and Christofides (2000a, b), Christofides and Armaou (2000) and Lunasin and Titi (2017). In Armaou and Christofides (2000a, b), a finite-dimensional controller was designed on the basis of a finite-dimensional system that captures the dominant (slow) dynamics of the infinite-dimensional system. In Christofides and Armaou (2000), the problem of global exponential stabilization of the KSE subject to periodic boundary conditions was considered. In Lunasin and Titi (2017), a distributed finite-dimensional feedback controller based on either point or averaged measurements of the state was proposed.

For practical application of finite-dimensional controllers for partial differential equations (PDEs), their sampled-data implementation is important. Sampled-data control of PDEs is becoming a hot topic. Sampled-data control of KSE was studied in Ghantasala and El-Farra (2012), where model reduction approach was suggested, and the design was based on the finite-dimensional system that captures the dominant dynamics. The latter approach is a qualitative one without giving explicit bounds on the performance (e.g. decay rate) or on the domain of attraction of the closed-loop system.

Distributed sampled-data control of PDEs under the point or spatially averaged measurements was suggested in Bar Am and Fridman (2014), Fridman and Bar Am (2013) and Fridman and Blighovsky (2012), where LMI conditions for the exponential stability and L_2-gain analysis of the closed-loop systems were derived in the framework of time-delay approach to sampled-data control by employing appropriate Lyapunov functionals. However, the above results were *confined to diffusion equations and to globally Lipschitz nonlinearities*, where stabilization is global. Distributed sampled-data control of various classes of PDEs is of great interest.

In the present paper, we introduce distributed sampled-data control of 1-D nonlinear KSE with the Dirichlet or periodic boundary conditions. The sensors provide either point or averaged

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discrete-time measurements of the state. The controllers enter KSE through distributed in space shape functions and the control signals are generated by zero-order hold devices. As for the diffusion equation, we exploit the time-delay approach to sampled-data control and the descriptor method for Lyapunov–Krasovskii-based delay-dependent stability analysis (Fridman, 2001, 2014; Fridman & Orlov, 2009). In terms of LMIs, we give regional exponential stability conditions for the sampled-data closed-loop system and find a bound on the domain of attraction (i.e. on the set of initial conditions, starting from which the solutions are exponentially converging). Under the corresponding continuous-time controllers, we derive LMI conditions for the global exponential stability. Some preliminary results under point state measurements will be presented in Kang and Fridman (2018).

The paper is organized as follows. Problem formulation is given in Section 2. In Sections 3 and 4, continuous in time and sampled-data controllers under the point or averaged state measurements are constructed to stabilize the system. Section 5 contains a numerical example to illustrate the efficiency of the main results. Finally, some concluding remarks are presented in Section 6 and some proofs are given in the Appendix.

Notation. $L^2(0, L)$ stands for the Hilbert space of square integrable scalar functions $u(x)$ on $(0, L)$ with the corresponding norm $\|u\|_{L^2} = [\int_0^L u^2(x)dx]^{\frac{1}{2}}$. The Sobolev space $H^k(0, L)$ is defined as

$$H^k(0, L) = \{u : D^\alpha u \in L^2(0, L),\ \forall\ 0 \le |\alpha| \le k\}$$

with norm $\|u\|_{H^k} = \left\{\sum_{0\le|\alpha|\le k} \|D^\alpha u\|^2_{L^2}\right\}^{\frac{1}{2}}$. Moreover,

$$H_0^k(0, L) = \{u \in H^k(0, L) : u(0) = Du(0) = \cdots = D^{k-1}u(0) = 0,\ u(L) = Du(L) = \cdots = D^{k-1}u(L) = 0\}.$$

2. Problem formulation and useful lemmas

We consider 1-D Kuramoto–Sivashinsky equation

$$u_t(x, t) + u_{xx}(x, t) + \nu u_{xxxx}(x, t) + u(x, t)u_x(x, t) = \sum_{j=1}^{N} b_j(x)U_j(t),\ 0 < x < L,\ t \ge 0, \tag{2.1}$$

subject to Dirichlet

$$u(0, t) = u(L, t) = 0,\ u_x(0, t) = u_x(L, t) = 0 \tag{2.2}$$

or to periodic

$$\frac{\partial^m u}{\partial x^m}(0, t) = \frac{\partial^m u}{\partial x^m}(L, t),\ m = 0, 1, 2, 3 \tag{2.3}$$

boundary conditions. Here ν is a positive constant, $u(x, t)$ is the state of KSE, and $U_j(t) \in \mathbb{R}, j = 1, 2, \ldots, N$ are the control inputs. Dirichlet boundary conditions were considered in Liu and Krstic (2001), whereas the periodic ones were studied in Armaou and Christofides (2000a, b) and Lunasin and Titi (2017). The open-loop system (2.1) (subject to $U_j(t) \equiv 0$) may become unstable if ν is small enough. Thus, for $L = 2\pi$ if $\nu < 1$ the open-loop system is unstable (see the example below).

As in Azouani and Titi (2014), Fridman and Bar Am (2013), Fridman and Blighovsky (2012) and Lunasin and Titi (2017), consider the points

$$0 = x_0 < x_1 < \cdots < x_N = L$$

that divide $[0, L]$ into N sampling intervals $\Omega_j = [x_{j-1}, x_j)$. Let

$$0 = t_0 < t_1 < \cdots < t_k \cdots,\quad \lim_{k\to\infty} t_k = \infty$$

be sampling time instants. The sampling intervals in time and in space may be variable but bounded,

$$0 \le t_{k+1} - t_k \le h,\ 0 < x_j - x_{j-1} = \Delta_j \le \Delta,$$

where h and Δ are the corresponding upper bounds. The control inputs $U_j(t)$ enter (2.1) through the shape functions

$$\begin{cases} b_j(x) = 1,\ x \in \Omega_j, \\ b_j(x) = 0,\ x \notin \Omega_j, \end{cases} j = 1, \ldots, N. \tag{2.4}$$

Sensors provide either point

$$y_{jk} = u(\bar{x}_j, t_k), \bar{x}_j = \frac{x_{j-1} + x_j}{2},\quad j = 1, \ldots, N,\ k = 0, 1, 2\ldots \tag{2.5}$$

or averaged

$$y_{jk} = \frac{\int_{x_{j-1}}^{x_j} u(x, t_k)dx}{\Delta_j},\ j = 1, \ldots, N,\ k = 0, 1, 2\ldots \tag{2.6}$$

measurements of the state. Our main objective is to design for (2.1) an exponentially stabilizing sampled-data controller that can be implemented by zero-order hold devices:

$$U_j(t) = -\mu y_{jk},\ j = 1, \ldots, N,\ t \in [t_k, t_{k+1}),\ k = 0, 1, \ldots, \tag{2.7}$$

where μ is a positive controller gain and y_{jk} is given by (2.5) or (2.6).

We formulate next some useful lemmas.[1]

Lemma 2.1 (*Poincaré Inequality Payne & Weinberger, 1960*). *Let $g \in H^1(0, L)$ be a scalar function with $\int_0^L g(x)dx = 0$. Then*

$$\int_0^L g^2(x)dx \le \frac{L^2}{\pi^2}\int_0^L \left[\frac{dg}{dx}(x)\right]^2 dx.$$

Lemma 2.2 (*Wirtinger Inequality and its Generalization Wang, 1994*). *Let $g \in H_0^1(0, L)$. Then the following inequality holds:*

$$\int_0^L g^2(x)dx \le \frac{L^2}{\pi^2}\int_0^L \left[\frac{dg}{dx}(x)\right]^2 dx.$$

Moreover, if $g \in H_0^2(0, L)$, then

$$\int_0^L \left[\frac{dg}{dx}(x)\right]^2 dx \le \frac{L^2}{\pi^2}\int_0^L \left[\frac{d^2g}{dx^2}(x)\right]^2 dx.$$

Lemma 2.3 (*Halanay's Inequality Halanay, 1966 or p.138 of Fridman, 2014*). *Let $0 < \delta_1 < 2\delta$ and let $V_1 : [t_0 - h, \infty) \to [0, \infty)$ be an absolutely continuous function that satisfies*

$$\dot{V}_1(t) \le -2\delta V_1(t) + \delta_1 \sup_{-h\le\theta\le 0} V_1(t + \theta),\ t \ge t_0.$$

Then

$$V_1(t) \le e^{-2\alpha(t-t_0)} \sup_{-h\le\theta\le 0} V_1(t_0 + \theta),\ t \ge t_0,$$

where α is a unique positive solution of

$$\alpha = \delta - \frac{\delta_1}{2}e^{2\alpha h}. \tag{2.8}$$

3. Continuous-time global stabilization

We will start with continuous in time results, where global stabilization can be achieved. Here the stability analysis is similar to Lunasin and Titi (2017), but differently from Lunasin and Titi (2017) we give a bound on the decay rate. Sampled-data controllers under the point/averaged measurements leading to regional stability will be presented in Section 4.

[1] It should be noted that the first Wirtinger's inequality in Lemma 2.2 is the one-dimensional Poincaré's inequality in Lemma 2.1 with optimal constant. This can be easily proved by the minimization principle of the nth eigenvalue.