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Active Disturbance Rejection Control: from ODEs to PDEs ⋆

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Abstract: This paper introduces a new emerging control technology, known as active disturbance rejection control to this day. We start its main idea and two main parts, namely, extended state observer and extended state observer based feedback for lumped parameter systems, and then discuss its application to both state and output feedback stabilization for distributed parameter systems.

Keywords: Active disturbance rejection control, feedback stabilization, PDEs

1. INTRODUCTION

Disturbance rejection is a different paradigm in control theory since the inception of the modern control theory in the later years of 1950's, seeded in Tsien (1954) where it is stated that the control operation "must not be influenced by internal and external disturbances" (Tsien, 1954, p.228). The tradeoff between mathematical rigor by model-based control theory and practicability by model-free engineering applications has been a constantly disputed issue in control community. On the one hand, we have mountains of papers, books, monographes published every year, and on the other hand, the control engineers are nowhere to find, given the difficulty of building (accurate) dynamic model for the system to be controlled, a simple, model free, easy tuning, better performance control technology more than proportional-integral-derivative (PID) control (Silva et al. (2002), see also Bialkowski et al. (2015)). This awkward coexistence of huge modern control theories on the one hand and a primitive control technology that has been dominating engineering applications for one century on the other pushed Jingqing Han, a control scientist at the Chinese Academy of Sciences to propose active disturbance rejection control (ADRC), as an alternative of PID. This is because PID has the advantage of model free nature whereas most parts of modern control theory are based on mathematical models. By model-based control theory, it is hard to cross the boundaries such as time variance, nonlinearity, and uncertainty created mainly by the limitations of mathematics. However, there are some basic limitations for PID in practice to accommodate the liability in the digital processors according to Han (2009).

To address this problem, Han would seek solution from the seed idea of disturbance rejection imbedded in Tsien (1954). Consider stabilization for the following second order Newton system:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = f(x_1(t), x_2(t), d(t), t) + u(t), \\ y(t) = x_1(t), \end{cases} \tag{1.1}$$

where $u(t)$ is the control input, $y(t)$ is the measured output, $d(t)$ is the external disturbance, and $f(\cdot)$ is an unknown function which contains unmodelled dynamics of the system or most possibly, the internal and external disturbance discussed in Tsien (1954).

The total disturbance can certainly be nonlinear, time variant and many other forms. Han considered it just as a signal of time, which is reflected in the measured output and hence can possibly be estimated. Let $a(t) = f(x_1(t), x_2(t), d(t), t)$. Then system (1.1) becomes

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = a(t) + u(t), \\ y(t) = x_1(t). \end{cases} \tag{1.2}$$

A flash of insight arises (Han (1989)): system (1.2) is exactly observable because it is trivially seen that $(y(t), u(t)) \equiv 0, t \in [0,T] \Rightarrow a(t) = 0, t \in [0,T]; (x_1(0), x_2(0)) = 0$ (see, e.g., (Cheng et al., 2015, p.5, Definition 1.2)). This means that $y(t)$ contains all information of $a(t)$! *Why not use $y(t)$ to estimate $a(t)$?*, was perhaps the question in Han's mind. If we can, for instance, $y(t) \Rightarrow \hat{a}(t) \approx a(t)$, then we can cancel $a(t)$ by designing $u(t) = -\hat{a}(t) + u_0(t)$ and system (1.2) amounts to, approximately of course,

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = u_0(t), \\ y(t) = x_1(t). \end{cases} \tag{1.3}$$

The nature of the problem is therefore changed now. System (1.3) is just a linear time invariant system for which we have many ways to deal with it. This is likewise feedforward control yet to use output to "transform" the system first. In a different point of view, this part is called

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the "rejector" of disturbance (Gao (2015)). It seems that a further smarter way would be hardly to find anymore because the control $u(t) = -\hat{a}(t) + u_0(t)$ adopts a strategy of estimation/cancellation, much alike our experience in dealing with uncertainty in daily life. One can imagine and it actually is, one of the most energy saving control strategies as confirmed in Zheng and Gao (2012).

This paradigm-shift is revolutionary for which Han wrote in Han (1989) that "to improve accuracy, it is sometimes necessary to estimate $a(t)$ but it is not necessary to know the nonlinear relationship between $a(t)$ and the states variables". The idea breaks down the garden gates from time varying dynamics (e.g., $f(x_1, x_2, d, t) = g_1(t)x_1 + g_2(t)x_2$), nonlinearity (e.g., $f(x_1, x_2, d, t) = x_1^2 + x_2^3$), and "internal and external disturbance" (e.g., $f(x_1, x_2, d, t) = x_1^2 + x_2^2 + \Delta f(x_1, x_2) + d$). The problem now becomes: how can we realize $y(t) \Rightarrow \hat{a}(t) \approx a(t)$?

Han told us in Han (1995) that it is not only possible but also realizable systematically. This is made possible by what is called extended state observer (ESO). Firstly, Han considered $a(t)$ to be an extended state variable and changed system (1.2) to

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = a(t) + u(t), \\ \dot{a}(t) = a'(t), \\ y(t) = x_1(t). \end{cases} \tag{1.4}$$

A linear observer for system (1.4), or equivalently linear ESO for system (1.2) can be designed as

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + a_1(\hat{x}_1(t) - y(t)), \\ \dot{\hat{x}}_2(t) = \hat{x}_3(t) + u(t) + a_2(\hat{x}_1(t) - y(t)), \\ \dot{\hat{x}}_3(t) = a_3(\hat{x}_1(t) - y(t)), \end{cases} \tag{1.5}$$

where we can choose high gains

$$a_i = \frac{\alpha_i}{\varepsilon^i}, i = 1, 2, 3, \tag{1.6}$$

so that

$$\begin{array}{l} \hat{x}_1(t) \to x_1(t), \hat{x}_2(t) \to x_2(t), \\ \hat{x}_3(t) \to a(t) \text{ as } t \to \infty, \varepsilon \to 0. \end{array} \tag{1.7}$$

The constants α_i in (1.6) are required to make

$$E = \begin{pmatrix} \alpha_1 & 1 & 0 \\ \alpha_2 & 0 & 1 \\ \alpha_3 & 0 & 0 \end{pmatrix} \tag{1.8}$$

be Hurwitz (Zheng et al. (2007); Guo and Zhao (2011)) and $a'(t)$ is required to be bounded. It is seen that we have obtained estimation $\hat{x}_3(t) \approx a(t)$ from $y(t)$!

Definition 1.1. The ESO (1.5) is said to be convergent, if for any given $\delta > 0$, there exist $T_\delta > 0, \varepsilon_\delta$ such that

$$\begin{array}{l} |\tilde{x}_i(t)| = |\hat{x}_i(t) - x_i(t)| \le \delta, \\ |\tilde{a}(t)| = |\hat{x}_3(t) - a(t)| \le \delta, \ \forall\, t > T_\delta, \varepsilon > \varepsilon_\delta, i = 1, 2. \end{array}$$

Finally, to stabilize system (1.2), we simply cancel the disturbance by using the ESO-based feedback:

$$u(t) = -\hat{x}_3(t) + \beta_1 \hat{x}_1(t) + \beta_2 \hat{x}_2(t), \tag{1.9}$$

where the first term is used to cancel (compensate) the disturbance and the last terms are stabilizing state feedback chosen by separation principle, i.e.

$$F = \begin{pmatrix} 0 & 1 \\ \beta_1 & \beta_2 \end{pmatrix} \tag{1.10}$$

is Hurwitz. The closed-loop of (1.2) under the feedback (1.9) becomes

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = a(t) - \hat{x}_3(t) + \beta_1 \hat{x}_1(t) + \beta_2 \hat{x}_2(t), \\ \dot{\hat{x}}_1(t) = \hat{x}_2(t) + a_1(\hat{x}_1(t) - y(t)), \\ \dot{\hat{x}}_2(t) = \hat{x}_3(t) - \hat{a}(t) + \beta_1 \hat{x}_1(t) + \beta_2 \hat{x}_2(t) \\ \qquad\qquad + a_2(\hat{x}_1(t) - y(t)), \\ \dot{\hat{x}}_3(t) = a_3(\hat{x}_1(t) - y(t)), \end{cases} \tag{1.11}$$

which is equivalent, by setting $\tilde{x}_1(t) = \hat{x}_1(t) - x_1(t)$, $\tilde{x}_2(t) = \hat{x}_2(t) - x_2(t)$ and $\tilde{a}(t) = \hat{x}_3(t) - a(t)$, to

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = \beta_1 x_1(t) + \beta_2 x_2(t) + \beta_1 \tilde{x}_1(t) + \beta_2 \tilde{x}_2(t) - \tilde{a}(t), \\ \dot{\tilde{x}}_1(t) = \tilde{x}_2(t) + a_1 \tilde{x}_1(t), \\ \dot{\tilde{x}}_2(t) = \tilde{a}(t) + a_2 \tilde{x}_1(t), \\ \dot{\tilde{a}}(t) = a_3 \tilde{x}_1(t) - a'(t). \end{cases} \tag{1.12}$$

Since $(\tilde{x}_i(t), \tilde{a}(t)) \to 0$ $i = 1, 2$ as $\varepsilon \to 0$ and $t \to \infty$, proved in convergence of ESO, we have immediately that

$$x_i(t) \to 0, i = 1, 2, \text{ as } t \to \infty, \varepsilon \to 0,$$

or equivalently

$$\begin{array}{l} x_i(t) \to 0, \hat{x}_i(t) \to 0, i = 1, 2, \\ \hat{x}_3(t) - a(t) \to 0 \text{ as } t \to \infty, \varepsilon \to 0. \end{array} \tag{1.13}$$

This is the well known separation principle in linear system theory. So, the whole idea not only works and but also works in an extremely wise way of estimating and cancelling the disturbance in real time.

Remark 1.1. System (1.1) is equivalent to second order system:

$$\ddot{x}(t) = f(x(t), \dot{x}(t), d(t), t) + u(t).$$

So the total disturbance and control are matched naturally. If they are not matched, for instance, system like

$$\begin{cases} \dot{x}_1(t) = x_2(t) + d(t), \\ \dot{x}_2(t) = u(t), \\ y(t) = x_1(t), \end{cases} \tag{1.14}$$

we can still apply ADRC to deal with stabilization. Actually, let

$$\bar{x}_2(t) = x_2(t).$$

Then (1.14) becomes

$$\begin{cases} \dot{x}_1(t) = \bar{x}_2(t), \\ \dot{\bar{x}}_2(t) = \dot{d}(t) + u(t), \\ y(t) = x_1(t), \end{cases} \tag{1.15}$$

For stabilization, we can achieve

$$x_1(t) \to 0, \bar{x}_2(t) = x_2(t) + d(t) \to 0 \text{ as } t \to \infty.$$

Certainly, as any other methods, some limitations likely exist in an otherwise perfect setting of ESO in the sense:

- The high gain is resorted in ESO to suppress the effect of the derivative $a'(t)$ of the total disturbance in (1.12);
- the derivative $a'(t)$ of disturbance as shown in (1.12) is supposed to be bounded as well as from (1.4) where $a(t)$ is regarded as an extended state variable.