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Brief paper

Cooperative robust output regulation of linear uncertain multiple multivariable systems with performance constraint ☆

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ABSTRACT

This paper is to pursue a general investigation of cooperative robust output regulation for linear continuous-time multiple multivariable systems with unknown system parameters and unmodeled external disturbances. We show that, under standard minimum-phase and relative degree like assumptions, an *internal model principle* based output-feedback protocol can be constructed by incorporating suitable dynamic compensators, even when the parametric uncertainties are arbitrarily large in some sense. Moreover, we are able to establish a redesigned protocol by means of adapting the H_∞ control method. It assures a desired robustness property for the closed-loop system of attenuating external unmodeled disturbances. Hence, our study offers a performance-constrained robust control solution in a distributed control fashion.

1. Introduction

Cooperative control of multi-agent networks has drawn major attention in the past few years, booted in Jadbabaie, Lin, and Morse (2003) and Ren (2008) for the fundamental multi-agent single and double integrators. Recent developments have been substantially promoted to cope with a wider scope of homogeneous networks (see Ma, Li, and Zhang (2010), Wang, Cheng, and Hu (2008) and Zhang, Lewis, and Das (2011)), and more general heterogeneous networks (see Huang and Ye (2014), Isidori, Marconi, and Casadei (2014), Lunze (2012), Wang, Hong, Huang, and Jiang (2010), Wieland, Sepulchre, and Allgöwer (2011) and Zhang and Lewis (2012)). Further extended from the aforementioned literature, a promising research topic is to investigate the cooperative control problem of uncertain agents subject to external disturbances, which is of course of greater interest in practice.

Distributed H_∞ control has been extensively developed for the aforementioned cooperative control problems. It assures an attenuation property for dealing with unmodeled external disturbances. In this direction, Lin, Jia, and Li (2008) developed a matrix technique based controller, minimizing the H_∞ norm of the transfer function from disturbances to a certain average information for single-integrator networks. Oh, Moore, and Ahn (2014) and Zhao, Duan, Wen, and Chen (2012) addressed some significant extensions for higher-order homogeneous linear multi-agent networks, where the former proposed a state-feedback design and the latter developed an observer-based output-feedback design. Yaghmaie, Movric, Lewis, Su, and Sebek (2016) proposed a static output-feedback design for heterogeneous linear networks. Regarding the above results, it is noted that the performance measure gain is the H_∞ norm of a certain transfer function and is however limited since it cannot be made arbitrary small. On the contrary, Zhang, Saberi, Grip, and Stoorvogel (2015) proposed some distributed output-feedback design methods that are able to reduce the H_∞ norm at an arbitrary small level. From the aforementioned literature, it is worth noting that the developed protocols thereof have to rely on the precise knowledge of system parameters.

Toward cooperative control for agents undergoing uncertainties, the idea of robust control techniques has been extensively developed in the literature. It ensures a tolerance of permitting uncertainties ranging in a small compact region; see Trentelman, Takaba, and Monshizadeh (2013) and references thereof for recent advances. Taking different stands on treating uncertainties, some internal model principle based techniques have been proposed in

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the literature. Besides of treating parametric uncertainties, they can asymptotically reject those disturbances modeled by the leader in leader–follower multi-agent networks, see Wang et al. (2010) for a basic setting. Such general synthesis problem is also referred to as the cooperative robust output regulation problem. Wang et al. (2010) proposed a distributed p-copy internal model based approach for agents with structural uncertainties ranging over small neighborhoods of the nominal parameters. For up-to-date literature in this direction, we shall refer to Kim, Shim, and Seo (2011), Su (2016) and references therein. Specifically, several types of canonical internal model principle based protocols have been developed for networks comprising of single-input single-output (SISO) agents with large parametric uncertainties in some sense. From the previous literature, however, the unmodeled disturbances should be further dealt with to reach, to some extent, an acceptable internal model principle based protocol for general uncertain networks.

The objective of this paper is to explore a distinguished and representative scenario on cooperative robust output regulation control for linear multiple multivariable systems when both large parametric uncertainties and unmodeled uncertainties are encountered. The research interest is two-fold. First, we consider a distinguished situation of unknown system parameters or large parametric uncertainties arising in the leader–follower multivariable dynamic network. Second, besides the agent uncertainties, we further adapt a distributed robust H_∞ control method in a distributed control fashion, leading to certain improved robustness for cooperative control when external unmodeled disturbances are active. A preliminary version of this paper appeared in Wang, Su, and Xu (2017) without addressing the disturbance attenuation and H_∞ method.

Our main contribution is to develop an internal model approach for managing the situation of unknown system parameters relating to general networks comprising of multivariable agents, and to design a distributed robust H_∞ controller leading to a performance-constrained robust control solution for the problem. Distinguished from the usual linear control (essentially with small structural uncertainties to some extent) methods addressed in the past literature, a major technical challenge of the present study is that, when system parameters are unknown or undergo large parametric uncertainties, stabilizability in general does not lead to an acceptable linear control law. To surmount the above difficulty, we establish a crucial coordinate transformation for multivariable networks which gives rise to a specific multi-input multi-output (MIMO) output-feedback normal form. Hence, it enables us to achieve an internal model principle based cooperative robust output regulation design.

Organization: Section 2 introduces the problem formulation and technical assumptions. Section 3 elaborates the output-feedback design. Section 4 presents an illustrative example. Section 5 closes the paper. A self-contained result is put in Appendix for the readability.

Terminology: For a vector with m components $\xi_i = [\xi_{i,1}^{\mathsf{T}}, \ldots, \xi_{i,m}^{\mathsf{T}}]^{\mathsf{T}}$, we use $\xi_{i,k} \in \mathbb{R}^{r_i}$ for $1 \le k \le m$ to denote the kth component of ξ_i, and use $\xi_{i,k,l} \in \mathbb{R}$ for $1 \le l \le r_i$ to denote the lth element of the component $\xi_{i,k}$. The symbol $\otimes$ denotes the Kronecker product of two matrices. For a matrix $A \in \mathbb{R}^{n\times n}$, $A < 0$ (or > 0) means that A is negative (or positive) definite, and $\det A$ denotes the determinant of A. I_n denotes the n-dimensional identity matrix.

2. Formulation and hypothesis

This paper focuses on a representative uncertain linear time-invariant (LTI) network comprising of a group of multivariable agents described by

$$\dot{x}_i = \mathbf{A}_i(w)x_i + \mathbf{B}_i(w)u_i + \mathbf{E}_i(w)\varpi,\ \ y_i = \mathbf{C}_i(w)x_i,\ 1 \le i \le N \qquad (1)$$

with $x_i \in \mathbb{R}^{n_i}$ as the state, $y_i \in \mathbb{R}^p$ as the performance output, $u_i \in \mathbb{R}^m$ as the control input, w as the agent parametric uncertainty in a known compact set $\mathbb{W} \subset \mathbb{R}^{n_w}$ of arbitrarily large, and $\varpi \triangleq \varpi(t) \in \mathbb{R}^{n_\varpi}$ representing some unmodeled external disturbances that is time-varying and locally essentially bounded.[1] The matrices $(\mathbf{A}_i(w), \mathbf{B}_i(w), \mathbf{C}_i(w), \mathbf{E}_i(w)) \in \mathbb{R}^{n_i\times n_i} \times \mathbb{R}^{n_i\times m} \times \mathbb{R}^{p\times n_i} \times \mathbb{R}^{n_i\times n_\varpi}$ are continuous in w.

For the purpose of the cooperative robust output regulation design, we assume $m = p$ (note that, to assure solvability of the robust output regulation, the condition $m \ge p$ is necessary; see Davison (1976) for details), and configure a virtual leader to specify the desired collective output behavior or a prescribed pattern. Specifically, the leader with node index 0 for patterning desired outputs of the plant (1) is described by

$$\dot{x}_0 = \mathbf{A}_0 x_0,\ \ y_0 = \mathbf{C}_0 x_0 \qquad (2)$$

with $x_0 \in \mathbb{R}^{n_0}$ and $y_0 \in \mathbb{R}^m$ as the state and output, respectively. Also, to exclude the trivial cases, it is standing to assume that the eigenvalues of the matrix $\mathbf{A}_0$ have nonnegative real parts and the parameters of $\mathbf{A}_0$ are known.

Regarding the network (1) and (2), a digraph is further posed, namely interaction digraph denoted by a triplet $\mathcal{G} \triangleq \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$. Here $\mathcal{V} \triangleq \{0, 1, 2, \ldots, N\}$ is the node set, $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is the edge set with no self-loop and $(j, i) \in \mathcal{E}$ if and only if the controller of the ith agent can make use of the output of the jth agent, and $\mathcal{A} = [a_{ij}]_{i,j=0}^N$ is the weighted adjacency matrix satisfying that $a_{ij} \ge 0$ and $a_{ij} > 0$ if and only if $(j, i) \in \mathcal{E}$, $i, j \in \mathcal{V}$.

2.1. Problem statement

The preceding graph notation is henceforth used to form the local measurement output. Specifically, we define $e_i = y_i - y_0$, and $e_{vi} = \sum_{j=0}^N a_{ij}(y_i - y_j)$, $1 \le i \le N$, with e_i as the regulated output and e_{vi} as the measurement output. In fact, the latter one in practice is agent-wise available for the feedback design. It is easy to show that $e_v = [H \otimes I_m]e$, $e \triangleq [e_1^{\mathsf{T}}, \ldots, e_N^{\mathsf{T}}]^{\mathsf{T}}$, $e_v \triangleq [e_{v1}^{\mathsf{T}}, \ldots, e_{vN}^{\mathsf{T}}]^{\mathsf{T}}$, where $H = [h_{ij}]_{i,j=1}^N$ with $h_{ij} = \sum_{k=0}^N a_{ik}$ for $i = j$; $h_{ij} = -a_{ij}$ for $i \ne j$.

Control goal: *Consider the network comprising of (1) and (2). For any constant $\varepsilon > 0$, find a controller of the form*

$$\dot{\chi}_i = \mathbf{F}_{1i}\chi_i + \mathbf{F}_{2i}e_{vi},\ \ u_i = \mathbf{H}_{1i}\chi_i + \mathbf{H}_{2i}e_{vi} \qquad (3)$$

such that, the closed-loop system composed of (1) and (3) satisfies both the following conditions.

- *C1: The output regulation is achieved when $\varpi(t) \equiv 0$, i.e., the regulated output satisfies $\lim_{t\to+\infty} e(t) = 0$;*
- *C2: The evolution of the regulated output $e(t)$ satisfies the following specified performance constraint: for any $T > 0$,*

$$\int_0^T \|e(t)\|^2 \mathrm{d}t \le \varepsilon\Big[\int_0^T \|\varpi(t)\|^2 \mathrm{d}t + x^c(0)^{\mathsf{T}}\mathcal{Q}x^c(0)\Big] \qquad (4)$$

where $\mathcal{Q}$ is a specified positive semi-definite matrix and $x^c \triangleq [x_0^{\mathsf{T}}, x_1^{\mathsf{T}}, \chi_1^{\mathsf{T}}, \ldots, x_N^{\mathsf{T}}, \chi_N^{\mathsf{T}}]^{\mathsf{T}}$.

The above problem is called cooperative robust output regulation with H_∞ performance constraint and ε is referred to as the H_∞ performance measure from unmodeled external disturbances to regulated output. In particular, ε can be made arbitrarily small by turning design parameters of (3). It is noted that, the matrix $\mathcal{Q}$ is relevant to the designed controller as shown in the proofs of Proposition 3.1 and Theorem 3.1.

[1] A function $\varpi : [0, \infty) \to \mathbb{R}^{n_\varpi}$ is said to be locally essentially bounded, if the restriction of ϖ to every bounded subinterval of $[0, \infty)$ is bounded except for a set of Lebesgue measure zero, see Sontag (1998, pp. 470).