Automatica 94 (2018) 26–34

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Optimal evasive strategies for multiple interacting agents with motion constraints ☆

William Lewis Scott [a], Naomi Ehrich Leonard [b,*]

[a] Aerospace Engineering, University of Maryland, College Park, MD 20742, USA
[b] Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544, USA

ARTICLE INFO

Article history:
Received 14 November 2016
Received in revised form 17 February 2018
Accepted 23 March 2018

Keywords:
Autonomous mobile robots
Decentralized control
Differential games
Minimum-time control
Multi-agent systems
Optimal trajectory
Pursuit-evasion

ABSTRACT

We derive and analyze optimal control strategies for a system of pursuit and evasion with a single speed-limited pursuer, and multiple heterogeneous evaders with limits on speed, angular turning rate, and lateral acceleration. The goal of the pursuer is to capture a single evader in the minimum time possible, and the goal of each evader is to avoid capture if possible, or else delay capture for as long as possible. Optimal strategies are derived for the one-on-one differential game, and these form the basis of strategies for the multiple-evader system. We propose a pursuer strategy of *optimal target selection* which leads to capture in bounded time. For evaders, we prove how any evader not initially targeted can avoid capture. We also consider optimal strategies for agents with radius-limited sensing capabilities, proving conditions for evader capture avoidance through a local strategy of *risk reduction*. We show how evaders aggregate in response to a pursuer, much like animals behave in the wild.

1. Introduction

We consider a system with a single pursuer and multiple heterogeneous evader agents moving on the plane. The goal of the pursuer is to capture any *single* evader in the minimum time possible. The goal of each evader is to first of all avoid capture, and if that is not achievable to delay capture for as long as possible. The pursuer has limited speed, and the evaders have limits on speed, angular turning rate, and lateral acceleration. To analyze this system we apply the framework of *differential games* introduced by Isaacs (1965) and used to study pursuit and evasion, e.g., Başar and Oldser (1999), Elliott and Kalton (1972) and Pachter (1987). We examine optimal strategies for the one-on-one pursuit-evasion differential game under these motion constraints, and use those as building blocks for strategies in the system of multiple evaders.

In the multiple-evader system, we propose a strategy for the pursuer of *optimal target selection*, where the target is the evader that could be captured in minimum time in a one-on-one setting. For evaders, in the case of all-to-all sensing, we prove that any evader not currently the target can always choose from a set of *reactive evasion* control inputs in order to avoid capture. The currently targeted evader must use the optimal evasive strategy from the one-on-one game to delay its capture for as long as possible. We also consider the case in which the pursuer and evaders have radius-limited sensing and propose a local strategy of *risk reduction*. We prove that any evader that is not the target can avoid capture using the risk reduction strategy. The case with no constraint on turning rate is addressed in Scott and Leonard (2014).

Predator avoidance has long been considered a key factor in animal aggregation. The "selfish herd" of Hamilton (1971) is a seminal model of identical evaders on the plane. Hamilton showed that a group benefit is not necessary to explain aggregation; rather, a self-interested individual in a population stays close to others to reduce its own chance of being caught. We also consider a group of self-interested evaders, but we use continuous-time dynamics and heterogeneous evaders. We are also motivated by the problem of designing dynamics for group formation in engineered multi-agent systems. Our decentralized control law for a collective response to a moving threat accounts for practical motion constraints and provides a control mechanism for spontaneous aggregation.

Hamilton's model has been extended to include evolutionary dynamics and formation of large groups (Wood & Ackland, 2007). Numerical studies have examined properties of group motion in multiple-evader systems where biologically inspired strategies are chosen a priori: on the plane (Lee, Pak, & Chon, 2006), in discrete

☆ This research was supported in part by National Science Foundation grant ECCS-1135724 and Office of Naval Research grant N00014-14-1-0635. The material in this paper was partially presented at the 53rd IEEE Conference on Decision and Control, December 15–17, 2014, Los Angeles, CA, USA. This paper was recommended for publication in revised form by Associate Editor Michael M. Zavlanos under the direction of Editor Christos G. Cassandras.

* Corresponding author.
E-mail addresses: wlscott@umd.edu (W.L. Scott), naomi@princeton.edu (N.E. Leonard).

https://doi.org/10.1016/j.automatica.2018.04.008
0005-1098/

space (Vabø & Nøttestad, 1997), in three dimensions (Vabø & Skaret, 2008), with multiple pursuers (Angelani, 2012), and based on observations of crabs and shorebirds (Viscido, Miller, & Wethey, 2001). Non-spatially explicit game theoretic models of multiple-evader systems have been posed for both homogeneous evaders (Cressman & Garay, 2011), and heterogeneous evaders (Eshel, Sansone, & Shaked, 2006).

Cooperative evader strategies have been studied as differential games in systems where all evaders are captured in succession (Liu, Zhou, Tomlin, & Hedrick, 2013a, b), and in systems where evaders have defensive capabilities (Fuchs & Khargonekar, 2011). The problem of choosing the order in which to capture multiple evaders requires numerical optimization or approximate solutions for efficient computation. Because every evader will be captured and strategies are cooperative, evaders are driven apart rather than into aggregations, fundamentally different from the problem posed by Hamilton and studied in the present paper. Multiple pursuers against a single evader have been studied in many contexts (Bakolas, 2013; Bakolas & Tsiotras, 2010; Chen, Zha, Peng, & Gu, 2016; Selvakumar & Bakolas, 2016; Zhou et al., 2016), using tools such as generalized Voronoi diagrams. Oyler, Kabamba, and Girard (2016) analyzed a pursuit-evasion game on the plane in the presence of obstacles, using a time-to-reach partition to determine if two evaders can rendezvous with each other before capture by a single pursuer. Bakolas and Tsiotras (2012) considered a multi-pursuer system where the active pursuer is whoever can capture the single evader in minimum time. This is dual to the optimal target selection problem for the multiple-evader system.

We use a time-to-capture metric based on the solution to a one-on-one differential game to partition the plane into evader domains of danger in a multiple-evader system. The partition is useful both in the analysis of the pursuer strategy of optimal target selection, where the pursuer chooses its target based on which domain of danger it is in, and in each evader's strategy of reactive evasion to keep the pursuer from entering its own domain of danger.

Our analysis considers an "omnidirectional" pursuer with limited speed seeking to capture any single evader from a group of heterogeneous and non-cooperating evaders with limits on speed, angular turning rate, and lateral acceleration, motivated from legged locomotion. A study of the kinematics of horses during polo games (Tan & Wilson, 2011) indicates that grip strength and limb force limits constrain the maximum lateral acceleration during a turn. In the evader motion model, the limit on lateral acceleration serves to create a tradeoff between speed and maneuverability, as the agent cannot make a sharp turn while maintaining maximum speed.

Several recent papers examine differential games featuring steered agents with turning constraints, such as a differential-drive agent vs. an omnidirectional agent, each acting as pursuer and evader (Ruiz & Murrieta-Cid, 2016), and an omnidirectional pursuer vs. a car-like evader (Exarchos, Tsiotras, & Pachter, 2015). These types of dynamics have also been studied in minimum-time problems for a single agent, for the fixed-speed *Reeds-Shepp* vehicle (Sussmann & Tang, 1991), for a differential-drive vehicle with limited wheel speed (Balkcom & Mason, 2002), and in our own work on an agent with limited speed, turning rate, and lateral acceleration (Scott & Leonard, 2018). A biologically inspired analysis of pursuit and evasion with acceleration constraints by Howland (1974) suggests that a more agile but slower evader can escape from a fast pursuer with limited lateral acceleration by veering to the side at the last moment. Studies of evasive behavior in different animal species are reviewed in Domenici and Ruxton (2015).

Our major contributions are threefold. First, we prove an optimal strategy for a pursuer that seeks to capture, in minimum time, any single evader among multiple heterogeneous evaders moving in the plane with limits on speed, angular turning rate, and lateral acceleration. The strategy relies on the optimal solution to the corresponding one-on-one differential game, which is new relative to the literature due to the constraints imposed on the evader's motion. For the multiple-evader system, the pursuer will target one evader at a time but will switch to target another evader if and when the pursuer estimates that the other evader can be caught in the shortest time remaining. Second, we prove a reactive evasive strategy for each non-targeted evader that keeps it from becoming the target. The evasion strategies do not require cooperation and each non-targeted evader can stay close to the group and conserve energy while still avoiding capture. Third, we generalize our results to the system in which the pursuer and each evader has a limited sensing region. In this case, before using reactive evasion, each non-targeted evader responds with a risk reduction phase to decrease its chances of becoming the target. We show how each non-targeted evader will move closer to another with a lower speed limit, thus providing a distributed control mechanism for aggregation.

We define the problem and system equations in Section 2. In Section 3 we derive optimal trajectories and an evader feedback-control law for the one-on-one differential game with motion constraints. In Section 4 we prove the optimal strategies for the multiple-evader system. We introduce limits on sensing radius in Section 5 and examine evader risk reduction. We conclude in Section 6.

2. Problem statement and equations of motion

We consider a system on the plane with a single pursuer agent P and a heterogeneous group of n evader agents E_i. The pursuer P is modeled as an agent that can freely move in any direction with maximum speed $\bar{v}_p$, position $\mathbf{r}_p(t) \in \mathbb{R}^2$ at time t, and velocity control input $\mathbf{u}_p(t) = (v_{x_p}(t), v_{y_p}(t))^T \in \mathbb{R}^2$ with $\|\mathbf{u}_p(t)\|_2 \leq \bar{v}_p$ for all t. Evaders are modeled as steered agents with inputs of speed $v_i(t) \in \mathbb{R}$ and turning rate $\omega_i(t) \in \mathbb{R}$, written as $\mathbf{u}_i(t) = (v_i(t), \omega_i(t))^T$. An evader's state at time t is its position $\mathbf{r}_i(t) \in \mathbb{R}^2$ and its heading angle $\theta_i(t) \in \mathbb{S}^1$.

For each evader agent E_i, we impose the following motion constraints:

- *Forward motion:* Speed must satisfy $v_i(t) \geq 0$ for all time t, such that the agent never moves in reverse.
- *Limited speed:* Let $\bar{v}_i > 0$ be the maximum speed. The speed control must satisfy $v_i(t) \leq \bar{v}_i$ for all time t.
- *Limited turning rate:* Let $\bar{\omega}_i > 0$ be the maximum turning rate. The turning control must satisfy $|\omega_i(t)| \leq \bar{\omega}_i$ for all time t.
- *Limited lateral acceleration:* Let μ_i represent the maximum lateral acceleration (turning traction limit). The inputs $v_i(t)$ and $\omega_i(t)$ must satisfy $|v_i(t)\omega_i(t)| \leq \mu_i$ for all time t. We further impose the condition that $\mu_i < \bar{v}_i\bar{\omega}_i$ so that the lateral acceleration constraint is active on part of the boundary of the control domain.

We define the evader admissible control region $\Omega_{e_i} = \{\mathbf{u} = (v, \omega) \in \mathbb{R}^2 \,|\, 0 \leq v \leq \bar{v}_i, |\omega| \leq \bar{\omega}_i, |v\omega| \leq \mu_i < \bar{v}_i\bar{\omega}_i\}$ and the pursuer admissible control region $\Omega_p = \{\mathbf{u}_p \in \mathbb{R}^2 \,|\, \|\mathbf{u}_p\|_2 \leq \bar{v}_p\}$. Admissible controls $\mathcal{U}_i$ for evader E_i are bounded Lebesgue measurable functions from $\mathbb{R}_+$ to Ω_{e_i} and $\mathcal{U}_p$ for the pursuer from $\mathbb{R}_+$ to Ω_p.

The system equations of motion are

$$\begin{aligned} \dot{\mathbf{r}}_p &= \mathbf{u}_p, & \mathbf{u}_p &\in \mathcal{U}_p \\ \dot{\mathbf{r}}_i &= \begin{pmatrix} v_i \cos\theta_i \\ v_i \sin\theta_i \end{pmatrix}, & (v_i, \omega_i) &\in \mathcal{U}_{e_i} \\ \dot{\theta}_i &= \omega_i, \quad \text{for } i = 1, 2, \ldots, n. \end{aligned} \tag{1}$$