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Dynamic virtual holonomic constraints for stabilization of closed orbits in underactuated mechanical systems[☆]

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ABSTRACT

This article investigates the problem of enforcing a virtual holonomic constraint (VHC) on a mechanical system with degree of underactuation one while simultaneously stabilizing a closed orbit on the constraint manifold. This problem, which to date is open, arises when designing controllers to induce complex repetitive motions in robots. In this paper, we propose a solution which relies on the parameterization of the VHC by the output of a double integrator. While the original control inputs are used to enforce the VHC, the control input of the double-integrator is designed to asymptotically stabilize the closed orbit and make the state of the double-integrator converge to zero. The proposed design is applied to the problem of making a PVTOL aircraft follow a circle on the vertical plane with a desired speed profile, while guaranteeing that the aircraft does not roll over for suitable initial conditions.

Virtual holonomic constraints (VHCs) have been recognized to be key to solving complex motion control problems in robotics. There is an increasing body of evidence from bipedal robotics (Grizzle, Abba, & Plestan, 2001; Grizzle, Chevallereau, Sinnet, & Ames, 2014; Westervelt, Grizzle, Chevallereau, Choi, & Morris, 2007), snake robot locomotion (Mohammadi, Rezapour, Maggiore, & Pettersen, 2016), and repetitive motion planning (Ahmed, Hably, & Bacha, 2013; Shiriaev, Freidovich, Robertsson, Johansson, & Sandberg, 2007) that VHCs constitute a new motion control paradigm, an alternative to the traditional reference tracking framework. The key difference with the standard motion control paradigm of robotics is that, in the VHC framework, the desired motion is parametrized by the states of the mechanical system, rather than by time.

Geometrically, a VHC is a subset[1] of the configuration manifold of the mechanical system. Enforcing a VHC means stabilizing the subset of the state space of the mechanical system where the generalized coordinates of the mechanical system satisfy the VHC, while the generalized velocity is tangent to the VHC. This subset is called the *constraint manifold*.

Grizzle and collaborators (see, e.g., Westervelt et al., 2007) have shown that the enforcement of certain VHCs on a biped robot leads, under certain conditions, to the orbital stabilization of a hybrid closed orbit corresponding to a repetitive walking gait. The orbit in question lies on the constraint manifold, and the mechanism stabilizing it is the dissipation of energy that occurs when a foot impacts the ground. In a mechanical system without impacts, this stabilization mechanism disappears, and the enforcement of the VHC alone is insufficient to achieve the ultimate objective of stabilizing a repetitive motion. Some researchers (Shiriaev, Freidovich, & Gusev, 2010; Shiriaev, Perram, & Canudas-de-Wit, 2005) have addressed this problem by using the VHC exclusively for motion planning, i.e., to find a desired closed orbit. Once a suitable closed orbit is found, a time-varying controller is designed by linearizing the control system along the orbit. In this approach, the constraint manifold is not an invariant set for the closed-loop system, and thus the VHC is not enforced via feedback.

To the best of our knowledge, for mechanical control systems with degree of underactuation one, the problem of simultaneous enforcement of a VHC and orbital stabilization of a closed orbit lying on the constraint manifold is still open. The challenge in addressing this problem lies in the fact that the dynamics of the mechanical control system on the constraint manifold are unforced. Therefore, any feedback that asymptotically stabilizes the desired closed orbit cannot render the constraint manifold invariant, and

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[1] More precisely, an embedded submanifold.

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thus cannot enforce the VHC. To overcome this difficulty, in this paper we propose to render the VHC *dynamic*. By doing that, under suitable assumptions it is possible to stabilize the desired closed orbit while simultaneously enforcing the dynamic VHC.

Contributions of the paper. This paper presents the first solution of the simultaneous stabilization problem just described for mechanical control systems with degree of underactuation one. Leveraging recent results in Mohammadi, Maggiore, and Consolini (2017), we consider VHCs that induce Lagrangian constrained dynamics. The closed orbits on the constraint manifold are level sets of a "virtual" energy function. We make the VHC dynamic by parametrizing it by the output of a double-integrator. We use the original control inputs of the mechanical system to stabilize the constraint manifold associated with the dynamic VHC, and we use the double-integrator input to asymptotically stabilize the selected orbit on the constraint manifold. Because the output of the double-integrator acts as a perturbation of the original constraint manifold, we also make sure that the state of the double-integrator converges to zero. To achieve these objectives, we develop a novel theoretical result giving necessary and sufficient conditions for the exponential stabilizability of closed orbits for control-affine systems.

The benefits associated with the simultaneous stabilization proposed in this paper are as follows. First, in the proposed framework one may assign the speed of convergence of solutions to the constraint manifold independently of the orbit stabilization mechanism. In particular, one may enforce the dynamic VHC arbitrarily fast,[2] so that after a short transient, the qualitative behaviour of trajectories of the closed-loop system is determined by the dynamic VHC. Second, since the constraint manifold is asymptotically stable for the closed-loop system, trajectories originating near the constraint manifold remain close to it thereafter. From a practical standpoint, the two features just highlighted mean that the dynamic VHC offers some control over the transient behaviour of the closed-loop system. The simultaneous stabilization of the closed orbit means that, without violating the dynamic VHC, an extra stabilization mechanism makes the trajectories of the closed-loop system converge to the closed orbit.

The property just described is illustrated in this paper with an example, the model of a PVTOL aircraft moving along a unit circle on the vertical plane. The control specification is to make the aircraft traverse the circle with bounded speed, while guaranteeing that the aircraft does not undergo full revolutions along its longitudinal axis. In this context, the VHC constrains the roll angle of the aircraft as a function of its position on the circle, preventing the aircraft from rolling over. On the other hand, the simultaneous stabilization of the closed orbit corresponds to stabilizing a desired periodic speed profile on the circle *without violating the constraint.* The double-integrator state perturbs the constraint so as to induce the orbit stabilization mechanism.

Relevant literature. Previous work employs VHCs to stabilize desired closed orbits for underactuated mechanical systems (Canudas-de Wit, 2004; Canudas-de Wit, Espiau, & Urrea, 2002; Freidovich et al., 2009; Shiriaev et al., 2005). Canudas-de-Wit and collaborators (Canudas-de Wit et al., 2002) propose a technique to stabilize a desired closed orbit that relies on enforcing a virtual constraint and on dynamically changing its geometry so as to impose that the reduced dynamics on the constraint manifold match the dynamics of a nonlinear oscillator. In Canudas-de Wit (2004) and Shiriaev et al. (2005), Canudas-de-Wit, Shiriaev, and collaborators employ VHCs to aid the selection of closed orbits of underactuated mechanical systems. It is demonstrated that an unforced second-order system possessing an integral of motion describes the constrained motion. Assuming that this unforced system has a closed orbit, a linear time-varying controller is designed that yields exponential stability of the closed orbit. With the exception of Canudas-de Wit et al. (2002), the papers above do not guarantee the invariance of the VHC for the closed loop system. The idea of event-triggered dynamic VHCs has appeared in the work by Morris and Grizzle in Morris and Grizzle (2009) where the authors construct a hybrid invariant manifold for the closed-loop dynamics of biped robots by updating the VHC parameters after each impact with the ground. This approach is similar in spirit to the one presented in this paper. Finally, the paper (Čelikovskỳ & Anderle, 2016) discusses collocated VHCs, i.e., VHCs parametrized by actuated variables. In Section 6, we discuss the differences between the method presented in this article and the ones in Canudas-de Wit (2004), Canudas-de Wit et al. (2002) and Shiriaev et al. (2010, 2005). We also discuss the conceptual similarities between the method presented in this article and the one in Morris and Grizzle (2009).

Organization. This article is organized as follows. We review preliminaries in Section 1. The formal problem statement and our solution strategy are presented in Section 2. In Section 3 we present dynamic VHCs. In Section 4 we present a novel result of a general nature providing necessary and sufficient conditions for the exponential stabilizability of closed orbits for control-affine systems, and use it to design the input of the double-integrator to stabilize the closed orbit relative to the constraint manifold. In Section 5 we present the complete control law solving the VHC-based orbital stabilization problem. In Section 6 we discuss the differences between the method presented in this article and the ones in Canudas-de Wit (2004), Canudas-de Wit et al. (2002) and Shiriaev et al. (2005). Finally, in Section 7 we apply the ideas of this paper to a PVTOL aircraft path following problem.

Notation. If $x \in \mathbb{R}$ and $T > 0$, then x modulo T is denoted by $[x]_T$, and the set $\{[x]_T : x \in \mathbb{R}\}$ is denoted by $[\mathbb{R}]_T$. This set can be given a manifold structure which makes it diffeomorphic to the unit circle $\mathbb{S}^1$. If a and b are vectors, then $\mathrm{col}(a, b) := [a^\top\ b^\top]^\top$. If $a, b \in \mathbb{R}^n$, we denote $\langle a, b\rangle = a^\top b$, and $\|a\| = \langle a, a\rangle^{1/2}$. If $A \in \mathbb{R}^{n\times n}$, we denote by $\|A\|_2$ the induced two-norm of A. If $(\mathcal{X}, d)$ is a metric space, Γ is a subset of $\mathcal{X}$, and $x \in \mathcal{X}$, we denote by $\|x\|_\Gamma$ the point-to-set distance of x to Γ, defined as $\|x\|_\Gamma := \inf_{y\in\Gamma} d(x, y)$. If $h : M \to N$ is a smooth map between smooth manifolds, and $q \in M$, we denote by $dh_q : T_qM \to T_{h(q)}N$ the derivative of h at q (in coordinates, this is the Jacobian matrix of h evaluated at q), and if M has dimension 1, then we may use the notation $h'(q)$ in place of dh_q. If M_1, M_2, N are smooth manifolds and $f : M_1 \times M_2 \to N$ is a smooth function, then $\partial_{q_1} f(q_1, q_2)$ denotes the derivative of the map $q_1 \mapsto f(q_1, q_2)$ at q_1. If $f : M \to TM$ is a vector field on M and $h : M \to \mathbb{R}^m$ is C^1, then $L_f h : M \to \mathbb{R}^m$ is defined as $L_f h(q) := dh_q f(q)$. For a function $h : M \to \mathbb{R}^m$, we denote by $h^{-1}(0) := \{q \in M : h(q) = 0\}$. If $A \in \mathbb{R}^{m\times n}$ has full row-rank, we denote by $A^\dagger$ the pseudoinverse of A, $A^\dagger = A^\top(AA^\top)^{-1}$. Given a C^2 scalar function $f : \mathbb{R}^n \to \mathbb{R}$, we denote by $\mathrm{Hess}(f)$ its Hessian matrix.

1. Preliminaries

Consider the underactuated mechanical control system

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + \nabla P(q) = B(q)\tau, \tag{1}$$

where $q = (q_1, \ldots, q_n) \in \mathcal{Q}$ is the configuration vector with q_i either a displacement in $\mathbb{R}$ or an angular variable in $[\mathbb{R}]_{T_i}$, with $T_i > 0$. The configuration space $\mathcal{Q}$ is, therefore, a generalized cylinder. In (1), $B : \mathcal{Q} \to \mathbb{R}^{n\times n-1}$ is C^1 and of full rank $n-1$. Also, $D(q)$, the inertia matrix, is positive definite for all q, and $P(q)$, the potential energy function, is C^1. We assume that there is a C^1 function $B^\perp : \mathcal{Q} \to \mathbb{R}^{1\times n}\setminus\{0\}$ such that $B^\perp(q)B(q) = 0$ for all $q \in \mathcal{Q}$.

[2] Naturally, actuator saturation will limit the maximum attainable speed of convergence to the constraint manifold.